Supervisory Incentives in a Banking Union*

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Abstract
We explore the behavior of supervisors in a “hub-and-spokes” regime: one in which a supranational agency has legal power over all decisions regarding banks, but has to rely on local supervisors to collect the information necessary to act. This institutional design entails a principal-agent problem between the central and local supervisors to the extent that their objective functions differ. Information collection will be inferior to what would happen in a model with fully independent local supervisors or one where the centralized agency directly collects information. The reason is that local agents will, in some states of the world, prefer to remain ignorant rather than to potentially learn information that would lead the central supervisor to decisions that are against the local agents’ interests. This, in turn, may lead to poorer ex ante incentives for regulated banks.

1 Introduction
The global financial crisis laid bare the limitations of a financial architecture in which nation-bound supervisors oversaw increasingly integrated financial markets and institutions. Supervisory fragmentation hindered the monitoring and understanding of cross-border linkages before the crisis, and led to often locally-driven and globally-inefficient policy actions after the crisis started. Against this background, the nascent banking union in Europe represents an important step in the direction of limiting the negative effects of the existing fragmentation. Yet, the move to supranational supervision raises new questions about its internal governance, its relationship with local supervisors, and ultimately, the way it will affect the behavior of the financial institutions under its jurisdiction. And, while the need for and benefits from a more internationally integrated supervisory regime have been discussed at length (Schoenmaker, 2011, Obstfeld, 2014), its costs and challenges have received much less attention.

This paper is a theoretical exploration of the tensions inherent in a “hub-and-spokes” supervisory regime: one in which a supranational agency has legal power over all decisions regarding banks, but has to rely on local

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supervisors to collect the information necessary to act. In particular, we focus on how this institutional design affects supervisors’ incentives to collect information; and on how this, in turn, influences bank behavior. Our framework is inspired by the supervisory reform in Europe.¹

We model an economy in which banks, protected by limited liability and operating under asymmetric information, tend to take on excessive risk in the absence of effective supervision. As in many related models, banks are levered and do not take into account the losses they impose on depositors and debt holders (and taxpayers when deposits are insured) when they fail (e.g., Hellmann et al., 2000, Matutes and Vives, 2000, Repullo, 2004, and Dell’Ariccia and Marquez, 2006). In our framework, bank risk taking is not directly observable, and the informational asymmetries this engenders prevents investors from pricing risk at the margin. The result is more risk taking than is socially optimal. Bank supervision is designed to improve over this laissez-faire equilibrium.

Under independent supervision, local supervisors invest resources to collect information about bank risk taking and, upon obtaining it, they can intervene in a bank and force it to invest in the portfolio deemed optimal by the supervisor, which is typically a safer portfolio. To maximize the disciplining effect of a possible intervention, we assume that when intervention takes place the bank’s shareholders are fully expropriated, which can be interpreted as putting an excessively risky bank under receivership or, at the extreme, nationalizing it. Intervention, however, comes at a cost. This can be seen as a reputational cost for the supervisor, the loss associated with the removal of a national (and private) champion, and/or it could represent a loss in efficiency associated with the transfer of the bank to the public sector.

Under centralized supervision, local supervisors retain control of information collection, but are mandated to transmit to the central agency what they learn. Then, the central supervisor can act on the information and has full control over the decision of whether or not to intervene a bank, and what portfolio to implement conditional on intervention. Critically, local supervisors have utility functions that are different, perhaps just slightly, from that of the central agency, and are in general less inclined to intervene in banks. Such reluctance to intervene may stem from greater costs that are borne at the local level for the supervisor, such as the aforementioned reputational costs and/or fiscal costs, or may reflect some degree of regulatory capture to which a central supervisor would not be subjected (see Agarwal et al., 2014, Acharya et al., 2013, and Bolton and Jeanne, 2011). This generates a principal-agent problem between the central and local

¹In its simplicity, our model cannot do justice to the many checks and balances and corrective procedures existing in a real-world supervisory mechanism. Rather, the model is meant to highlight some of the tensions that the new supervisory regime will have to take into account in order to operate effectively.
supervisors, in addition to that between supervisors and banks, that is at the core of our model. Information collection will be distorted away from what would happen in a model with fully independent local supervisors or one where the centralized agency directly collects information. The reason is that local agents will, in some states of the world, prefer to remain ignorant rather than to potentially learn information that would lead the central supervisor to decisions that are against the local agents’ interests.

This poorer information collection entails costs. The problem for the central agency is obvious. But the lack of information can also lead to results that are undesirable for local supervisors, and may lead to inefficient outcomes in terms of bank resolution. This, in turn, may lead to poorer ex ante incentives for regulated banks. A lower probability of having its actions discovered will make it more attractive for banks to take risk in excess of that desired by the regulator. That said, to the extent that a centralized supervisor imposes tighter standards (tolerates less risk taking) than local ones, for some banks this effect will be partly offset by a lower threshold for intervention.

Building on this result, the paper delves into what factors make the conflict more or less relevant and what policies the central supervisor can enact to correct it. The starting point of the analysis is that the local and the central supervisors have different utility functions and, consequently, can take different decisions. In other words, there are some states of the world for which the local supervisor would allow certain banks to operate while the central agency would prefer to intervene and resolve them. This stems from two main forces: First, banks may be systemic at the national but not the supranational level. Second, central supervisors internalize the cost of resolution, which may have negative externalities for other international institutions, more than the local agency. Based on this setup we expect the conflict to be greater for: 1) Regional banks that are systemic for individual countries but not for the broader banking union as a whole. 2) Local supervisors in fiscally weak countries that are more reluctant to bear the cost of resolution but that have internationally important banks. 3) Concentration: in small countries, a more concentrated banking system increases the probability of having locally systemic but not globally systemic banks. In large countries, this may not be true.

The paper is relevant for the nascent banking union in Europe. As of November 2014, the Single Supervisory Mechanism (SSM), which will reside within the European Central Bank, will be the primary supervisor of the Eurozone’s biggest banks. It will supervise directly the largest 128 banks in the Eurozone, accounting for approximately 85 percent of banking assets in the Eurozone, and indirectly all the banks in the Eurozone.
Yet, at least for a prolonged transition period, one can realistically assume that the SSM will have to rely heavily on local supervisors for the collection and processing of on-site information. Then, our paper suggests that internal mechanisms need to be devised to guarantee that the spokes, which may have different objective functions from the hub, act according to the centralized mandate. Various elements of the new institutional design go in this direction. For instance, the SSM retains the right to bring any bank (in addition to the top 128) under its direct supervision. In our model, this kind of threat would act as a discipline device for local regulators. The ECB also plans to appoint multi-country teams headed by SSM’s officials to conduct on-site inspections at the largest banks; again a move that facilitates the exchange of information between the spokes and the hub.

This paper contributes to the literature on the benefits and challenges of centralized bank regulation and supervision. In this respect the paper is related to Dell’Ariccia and Marquez (2006), Calzolari and Gyongyi (2011), Holthausen and Rønde (2004), Rishi et al. (2013), and the discussions in Basel Committee on Banking Supervision (2010) and IMF (2010).

The paper proceeds as follows. Section 1 describes the basic model with a local independent supervisor. Section 2 derives the bank’s portfolio choice in the case of no supervision. Section 3 derives the equilibrium in the case of an independent local supervisor, while Section 4 discusses the effects of centralizing supervision.

2 Model

Consider a simple one-period economy with banks, investors and a local supervisor. Each bank has access to a risky investment portfolio and needs external funds to finance it. The supervisor may observe the bank’s portfolio and decide to intervene in the bank or let it operate. Finally, the bank’s portfolio returns are realized and the bank either succeeds or fails.

Specifically, the bank’s investment portfolio requires 1 unit of funds. Each bank is endowed with an amount of capital \(k \in [0, 1]\) and raises an amount of deposits \(1 - k\) from outside investors. Capital is distributed across banks according to the cumulative distribution \(F(k)\), with density \(f(k)\).

The opportunity cost of capital is \(r_E \geq 1\) per unit. Depositors receive a promised (per unit) return \(r_D\) and have a total per unit (normalized) opportunity cost of 1. The deposit market is perfectly competitive so that the bank will always set \(r_D\) at the level required for depositors to recover their opportunity cost of funds and be willing to participate. For simplicity, we consider that deposits are fully insured so that \(r_D = 1\).
The bank chooses a portfolio on the efficient frontier, where that frontier is defined as follows: if a portfolio with probability of repayment $q$ is chosen, the return on the portfolio is $R - \frac{1}{2}cq$. In other words, the bank can choose the level of risk it likes, with the implication that a higher risk (i.e., lower $q$) portfolio has a higher return if successful, but repays less in case of success. This gives a familiar risk-return tradeoff.

The local supervisor may decide to inspect the bank and observe the quality of the bank's portfolio. Specifically, the supervisor chooses initially an inspection effort $e$ at a cost $e^2$, which represents the probability of observing the portfolio $q$ chosen by the bank, as well as the bank's level of capital (i.e., the supervisor observes the bank's balance sheet). Conditional on the result of the inspection, the supervisor decides whether to intervene in the bank or let it operate independently. If it intervenes, the bank is put under receivership. The supervisor takes control over the investment choice and chooses the investment portfolio that maximizes total social surplus. Existing bank shareholders are expropriated in that the supervisor obtains the returns from successful portfolios after repaying the promised returns to depositors. Intervention entails a cost $A_L$ to the supervisor.

Irrespective of whether it has intervened in the bank, the supervisor provides insurance to depositors in case the bank’s portfolio fails at the final stage. In other words, the supervisor repays depositors when, with probability $1 - q$, the bank’s investment returns 0 at the end of the period. Providing insurance is costly in that there is a (per unit) cost of funds $\psi_L \geq 1$. The cost $\psi_L$ can be thought of as the shadow cost associated with having to use some public funds to repay depositors. In other words, it represents the lost opportunity by the local supervisor to use the funds for another activity producing a benefit of $\psi_L$. Alternatively, it may represent the cost of externalities associated with bank failure, which grow with the size of the realized losses to bank depositors.

The timing of the model is as follow. In stage 0, a bank with capital $k$ chooses its investment portfolio and the local supervisor chooses the inspection effort $e$. In stage 1, with probability $e$, the supervisor observes $q$ and decide whether to intervene. If intervened, the bank is put under receivership and the supervisor’s preferred investment portfolio is chosen. If not intervened, the bank continues with its prior choice of portfolio quality $q$. In either case, in stage 3 the investment portfolio returns are realized, and depositors obtain their promised return $r_D$ either from the bank or the supervisor. Capital providers obtain the remaining profits from the bank’s portfolio after repaying depositors when the bank is not intervened and the portfolio succeeds. Otherwise, they obtain zero. In order for them to be willing to participate, shareholders must obtain at least
in expectation.

3 The bank’s investment choice in the case of no supervision

We start by considering the bank’s optimal investment in the case of autarky, that is when there is no supervisory inspection and no intervention. The bank chooses the quality of the investment portfolio \( q \) so as to maximize expected profits.

If the bank were fully equity financed, its objective function would be

\[
\max_q q \left( R - \frac{1}{2} cq \right) - r_E.
\]

The optimal choice of \( q \) solves

\[
R - cq = 0
\]

and thus equals

\[
q^* = \frac{R}{c}.
\]

We denote this as the first best investment portfolio since it internalizes the cost of failure (i.e., there is no risk shifting on to depositors). We assume \( R < c \) so that \( q^* < 1 \).

Now consider the case of a levered institution: The bank has some capital \( k \) and must finance the rest with deposits. The bank’s objective function is then

\[
\max_q q \left( R - \frac{1}{2} cq - (1 - k) \right) - kr_E,
\]

reflecting that the bank only repays its depositors, \( 1 - k \), when its portfolio repays, which occurs with probability \( q \). Thus, in autarky the bank chooses an investment portfolio of quality \( \hat{q}(k) \), where

\[
\hat{q}(k) = \frac{R - (1 - k)}{c}.
\] (1)

The bank chooses \( \hat{q}(k) < q^* \) when levered because of limited liability: the bank does not internalize the losses that accrue to depositors in case of failure. Note that the chosen quality of the investment portfolio is a function of the capital \( k \) a bank has and, in particular, it is increasing in \( k \). This implies that high capital banks will choose lower risk portfolios, and low capital banks will choose higher risk portfolios. Importantly, high capital banks will choose portfolios that are closer to the first best choice \( q^* \).

This framework implies a moral hazard problem in the choice of the investment portfolio quality when the bank raises a positive amount of deposits. The quality of the investment portfolio may be so low that the
supervisor may decide to intervene and take over control of the bank’s investment. This is precisely what we analyze next, starting with the case when there is only a local supervisor that chooses both the inspection effort and the intervention strategy. For simplicity we proceed first with the basic case where there is no differential cost between equity and debt financing, so that \( r_E = 1 \).

4 Equilibrium in the case of an independent local supervisor

In this section we consider the case where there is a local supervisor that chooses an inspection effort \( e \) and an intervention strategy, which amounts to a portfolio choice for the bank upon intervention. The model is solved backward. We first analyze the supervisor’s intervention decision conditional on its inspection having been successful, and then look at the choice of \( e \).

4.1 Intervention choice of the local supervisor

Upon learning the bank’s portfolio - asset allocation \( q \) and liabilities, including capital \( k \) - the supervisor chooses whether to intervene so as to maximize total surplus, which includes financial as well as possible non-pecuniary returns. Given a portfolio choice \( q \) by the bank, if the supervisor does not intervene, its payoff is given by

\[
q \left( R - \frac{1}{2} cq - (1 - k) \right) - (1 - q) (1 - k) \psi_L - k.
\]  

The first term reflects that with probability \( q \) the bank’s portfolio succeeds and produces a surplus \( R - \frac{1}{2} cq - (1 - k) \) after repaying 1 to the \((1 - k)\) depositors. The second term is the payoff when with probability \( 1 - q \) the bank’s portfolio fails and returns 0. In this case, the \( 1 - k \) of deposits are repaid through deposit insurance at the per unit cost \( \psi_L \). The last term is the capital \( k \) that is employed in the investment portfolio.

In case the bank is intervened, the supervisor’s payoff is instead given by

\[
-A_L + q_L^* \left( R - \frac{1}{2} cq_L^* - (1 - k) \right) - (1 - q_L^*) (1 - k) \psi_L - k,
\]  

where

\[
q_L^* = \frac{R + (1 - k) (\psi_L - 1)}{c}
\]  

is now the investment portfolio quality chosen by the supervisor to maximize its payoff in the case of intervention, i.e., it is the portfolio quality that maximizes (3). Note that \( q_L^* > q^* \) as the supervisor takes into account the greater cost of failure associated with the higher shadow cost \( \psi_L > 1 \) while a fully equity financed bank does not when choosing \( q^* \). Also, note that \( \frac{\partial q_L^*}{\partial k} < 0 \) since deposit insurance outlays conditional
on failure decrease with the bank’s capitalization. At the limit, if \( k = 1 \) so that the bank is all equity financed, \( q_L^* = q^* = \tilde{q} \), and there is no need for intervention.

The first term in (3), \(-A_L\), represents the cost of intervention for the social supervisor. The other two terms represent the net expected surplus from investing in a portfolio of quality \( q_L^* \). With probability \( q_L^* \) the investment yields a return net of depositors’ repayment of \( R - \frac{1}{2}cq_L^* - (1 - k) \), and with probability \( 1 - q_L^* \) it fails and the \( 1 - k \) deposits are repaid through deposit insurance at the per-unit cost \( \psi_L \). The last term is again the capital \( k \) employed in the bank’s investment.

Let \( I(q', q, k) \) indicate the difference between the intervention and no intervention payoffs for the supervisor when he implements \( q' \) under intervention, under no intervention the bank chooses \( q \), and the bank has a level of capital \( k \). Assuming the local supervisor chooses \( q_L^* \) when he intervenes, we have:

\[
I_L(q_L^*, q, k) = -A_L + q_L^* \left[ R - \frac{1}{2}cq_L^* - (1 - k) \right] - (1 - q_L^*) (1 - k) \psi_L
\]

\[
- \left( q \left[ R - \frac{1}{2}cq - (1 - k) \right] - (1 - q) (1 - k) \psi_L \right) .
\]

In what follows we assume \(-A_L + q_L^* \left( R - \frac{1}{2}cq_L^* - 1 \right) - (1 - q_L^*) \psi_L > 0 \) so that, at least for banks with zero capital, intervention does not entail any social loss. Moreover, we also assume that intervention is always desirable for banks with no capital. Formally, this means \( I_L(q_L^*, q, k) > 0 \) for \( k = 0 \). After substituting the expression for \( q_L^* \) into (5), this boils down to requiring that \( \psi_L > \sqrt{2cA_L} \). We have then the following immediate result.

**Proposition 1** When acting independently, the local supervisor chooses to intervene in a bank with capital \( k \) if \( q < \tilde{q}_L(k) \), where

\[
\tilde{q}_L(k) = \frac{1}{c} \left( R + (1 - k) (\psi_L - 1) - \sqrt{2cA_L} \right) ,
\]

It follows that \( \frac{\partial \tilde{q}_L(k)}{\partial k} < 0 \), \( \frac{\partial \tilde{q}_L(k)}{\partial \psi_L} > 0 \) and \( \frac{\partial \tilde{q}_L(k)}{\partial A_L} < 0 \).

**Proof.** Substituting the expression for \( q_L^* \) (k) as in (4) into (5) and solving it equal to zero gives

\[
\tilde{q}_L(k) = \frac{1}{c} \left( R + (1 - k) (\psi_L - 1) \pm \sqrt{2cA_L} \right) .
\]

We take the negative root for \( \tilde{q}_L \) as otherwise \( \tilde{q}_L(k) > q_L^*(k) \), which cannot be optimal given that it would imply an intervention threshold for \( q \) above the optimal portfolio quality chosen by the supervisor in case
of intervention. It is immediate to see that for $\psi_L > 1$, $\frac{\partial \bar{q}_k}{\partial k} = -\frac{1}{c} (\psi_L - 1) < 0$, $\frac{\partial \bar{q}_k}{\partial \psi_L} = \frac{1}{c} (1 - k) > 0$ and $\frac{\partial \bar{q}_k}{\partial A_L} = -\frac{1}{\sqrt{2cA_L}} < 0$. 

The proposition states that the local supervisor will intervene when the quality of the bank’s portfolio is too low. The intervention threshold $\bar{q}_L (k)$ is a function of a bank’s actual level of capital. In fact, $\bar{q}_L (k)$ defines a schedule of thresholds as a function of $k$, with $\tilde{q}_L$ decreasing in $k$. This implies that a supervisor is willing to be more lenient with better capitalized banks in the sense of allowing a bank with more capital to operate with a riskier portfolio. The reason is that the social cost of providing public funds in case of failure, $(1 - k) (\psi_L - 1)$, is lower the more capital the bank has. This leads the supervisor to be less inclined to intervene for more capitalized banks.\(^2\) This result reminds of risk-weighted capital requirements: banks with riskier portfolios are required to hold more capital. In practice, regulatory measures such as CARs are justified on the basis of the loss absorbing capacity of banks capital. The intuition for our result is similar: upon failure, better capitalized banks entail lower costs in terms of deposit insurance outlays.

It follows from the arguments above that, for a given level of capital $k$, the threshold $\bar{q}_L (k)$ increases with the supervisor’s shadow cost of funds $\psi_L$ so that an increase in $\psi_L$ makes the supervisor more prudent. This means that the prospect of more costly deposit insurance outlays in case of failure, or simply more costly externalities associated with failure, leads to a more aggressive “prompt corrective action” stance.

Finally, for any given level of capital $k$, the critical portfolio quality $\bar{q}_L$ below which a bank is intervened is lower than the supervisor’s choice of investment quality $q^*_L$ because of the intervention cost $A_L$. This implies that banks with investment portfolios of quality $q \in (\bar{q}_L, q^*_L]$ are not intervened. In the absence of the intervention cost, i.e., if $A_L = 0$, then $\bar{q}_L = q^*_L$. It follows that the supervisor becomes laxer in its intervention decisions as the cost $A_L$ increases.

Note that in this section we have characterized the subgame perfect intervention strategy (that is the strategy that will prevail when the supervisor cannot commit ex ante to a specific intervention threshold). Put differently, the strategy is ex-post optimal. But, ex-ante, the supervisor might prefer to commit to different (higher) intervention standards. As we will show in the next section, higher standards would lead to higher equilibrium supervisory effort and an overall better outcome for the supervisor.

\(^2\) As we show below, however, this will nevertheless translate into a unique capital threshold below which intervention occurs, and above which the bank is allowed to continue. The decreasing nature of $\tilde{q}_L$ with respect to $k$ does, however, influence bank behavior, as we discuss further below.
4.2 Bank portfolio choice

Now consider how the bank’s portfolio choice changes once we introduce the possibility of regulatory action.

For a choice of portfolio quality $q$, the bank’s profit function can be rewritten as

$$\Pi = \begin{cases} q \left( R - \frac{1}{2} cq - (1 - k) \right) - k & \text{for } q \geq \tilde{q}_L \\ (1 - e) q \left( R - \frac{1}{2} cq - (1 - k) \right) - k & \text{for } q < \tilde{q}_L. \end{cases}$$  \hspace{1cm} (7)$$

The first expression in (7) represents the expected profit for a bank with portfolio quality $q \geq \tilde{q}_L$. The second line is instead the profit for a bank with $q < \tilde{q}_L$, which obtains a positive payoff only when, with probability $1 - e$, it is not intervened and can continue the investment, returning $q \left( R - \frac{1}{2} cq - (1 - k) \right)$.

It is immediate to see that the threat of regulatory intervention may affect a bank’s portfolio choice. In particular, banks with a level of capital $k$ such that their laissez-faire optimal portfolio $\tilde{q}(k)$, as defined in (1), is greater than the intervention threshold $\tilde{q}_L(k)$, as defined in Proposition 1, will not be affected by the threat of regulatory intervention and will continue to choose their desired portfolio $\tilde{q}(k)$. By contrast, banks whose capital $k$ is such that $\tilde{q}(k) < \tilde{q}_L(k)$ will now have to take into account that, with probability $e$, they will be intervened and lose their franchises. This implies that some of these banks, and in particular those with a level of capital such that the difference $\tilde{q}_L(k) - \tilde{q}(k)$ is small enough, may now opt for the portfolio $\tilde{q}_L$ in order to avoid intervention.

To see this formally, we proceed in steps. We first recall that the regulatory threshold $\tilde{q}_L(k)$ for intervention decreases with $k$ while the laissez faire choice $\tilde{q}$ increases with $k$. This implies that the difference $\tilde{q}_L(k) - \tilde{q}(k)$ is decreasing in $k$ and there is a unique level of bank capital $k$, denoted $\tilde{k}_L$, at which $\tilde{q}_L(k) = \tilde{q}(k)$. Equating (1) and (6) and solving for $k$ gives

$$\tilde{k}_L = \left( 1 - \frac{1}{\psi_L} \sqrt{2cA_L} \right).$$ \hspace{1cm} (8)$$

Under the assumption that $\psi_L > \sqrt{2cA_L}$, which, as argued above, guarantees that intervention is at least sometimes optimal, we have $\tilde{k}_L \in (0, 1)$. It follows that banks with $k \geq \tilde{k}_L$ will continue finding it optimal to choose their laissez-faire portfolio $\tilde{q}(k)$ and won’t be intervened since this portfolio is already safer than the intervention threshold $\tilde{q}_L(k)$. These banks are thus unaffected by the presence of a supervisor. By contrast, banks with capital $k < \tilde{k}_L$, and thus $\tilde{q}(k) < \tilde{q}_L(k)$, will now compare their expected profit when choosing $\tilde{q}(k)$ with that when switching to $\tilde{q}_L(k)$. Using (7), the expected profit is given by

$$\Pi (\tilde{q}) \mid_{k < \tilde{k}_L} = (1 - e) \tilde{q} \left( R - \frac{1}{2} c \tilde{q} - (1 - k) \right) - k.$$ \hspace{1cm} (9)$$
in the former case and by

$$
\Pi(\tilde{q}_L)|_{k < \tilde{k}_L} = \tilde{q}_L \left( R - \frac{1}{2}c\tilde{q}_L - (1 - k) \right) - k
$$

(10)
in the latter. Define $D(k) = \Pi(\tilde{q}_L)|_{k < \tilde{k}_L} - \Pi(\tilde{q})|_{k < \tilde{k}_L}$. Then, clearly, a bank with a given level of capital $k < \tilde{k}_L$ will choose to switch to the supervisory intervention threshold $\tilde{q}_L(k)$ if $D(k) > 0$, and will stick to the laissez-faire portfolio $\hat{q}(k)$ otherwise. We have the following result.

**Lemma 2** For a given level of supervisory effort $e$, there exists a level of capital $\overline{k}_L(e)$ such that banks with $k < \overline{k}_L(e)$ choose the laissez-faire portfolio quality $\hat{q}(k)$, while banks with $k \geq \overline{k}(e)$ switch to a portfolio of quality $\tilde{q}_L(k)$, where

$$
\overline{k}_L(e) = \left( 1 - \frac{R\sqrt{e} + \sqrt{2cA_L}}{\psi_L + \sqrt{e}} \right) < \tilde{k}_L.
$$

It follows that $\frac{\partial \overline{k}_L(e)}{\partial e} < 0$, $\frac{\partial \overline{k}_L(e)}{\partial \psi_L} > 0$ and $\frac{\partial \overline{k}_L(e)}{\partial A_L} < 0$.

**Proof.** Substituting the expression for $\tilde{q}_L(k)$ as in (6) into (10) and that for $\tilde{q}$ as in (1) into (9) and solving $D(k) = \Pi(\tilde{q}_L)|_{k < \tilde{k}_L} - \Pi(\tilde{q})|_{k < \tilde{k}_L} = 0$ with respect to $k$ gives $\overline{k}_L(e)$ as in 11. Using the expression for $\overline{k}_L$ as in (8) it is easy to see that $\overline{k}_L(e) < \tilde{k}_L$. Moreover, from (11) we have $\frac{\partial \overline{k}_L(e)}{\partial e} = -\frac{\psi_L R\sqrt{e} - \sqrt{2cA_L}}{2\sqrt{e}(\psi_L + \sqrt{e})^2} < 0$, $\frac{\partial \overline{k}_L(e)}{\partial \psi_L} > 0$ and $\frac{\partial \overline{k}_L(e)}{\partial A_L} = -\frac{c}{\sqrt{2cA_L(\psi_L + \sqrt{e})^2}} < 0$. The proposition follows. 

The lemma shows that the threat of regulatory intervention affects the portfolio choice of banks with capital in an intermediate range, $k \in [\overline{k}_L(e), \tilde{k}_L)$, as it induces them to switch from their laissez-faire portfolio quality $\hat{q} < \tilde{q}_L(k)$ and meet the supervisor’s standards. The threshold level $\overline{k}_L(e)$ is decreasing in the supervisory effort $e$, meaning that a greater number of banks with even lower levels of capital will choose to meet the supervisory standards as $e$ increases.

Moreover, $\overline{k}_L(e)$ increases with the shadow cost of public funds $\psi_L$, while it decreases with the cost of regulatory intervention $A_L$. This means that a decrease in $\psi_L$ (or an increase in $A_L$) induces banks with even lower levels of capital to meet the supervisory standards. The reason for these results is that, for a given level of supervisory effort $e$, these parameters affect the intervention threshold $\tilde{q}_L(k)$, as stated in Proposition 1. When $\tilde{q}_L(k)$ decreases, as for example as a result of a decrease in $\psi_L$ or an increase in $A_L$, the supervisor is more lenient in that it allows banks with lower portfolio quality to continue. This will induce banks with even lower capital to choose $\tilde{q}_L(k)$ as the difference between this and their laissez-faire optimal $\hat{q}(k)$ is reduced.

To guarantee that supervisory intervention is meaningful in the model, in what follows we will restrict parameters so that the threshold $\overline{k}_L(e)$ is positive. This implies that there will always be banks with capital
$k < \bar{k}_L$ that will choose $\hat{q} < \bar{q}_L$ and will be intervened with probability $e$ and banks with $k \in [\underline{k}(e), \bar{k}_L)$ that will instead choose $\bar{q}_L$ to avoid being intervened. Assuming $\bar{k}(e) > 0$ requires the parameters $R, \psi_L, A_L$ and $c$ to be such that in equilibrium the supervisor will choose an inspection effort $e < \bar{e}_L$ where, by setting (11) equal to zero, we have

$$\bar{e}_L = \frac{(\psi_L - \sqrt{2cA_L})^2}{(R - 1)^2}.$$ (12)

Figure 1 summarizes graphically the relationship between the bank portfolio choice as a function of $k$ for a given supervisory effort $e$. As the figure illustrates, banks choose their laissez-faire portfolio quality $\hat{q}$ for $k < \bar{k}(e)$ and for $k > \bar{k}(e)$. In the former case, $\hat{q}$ is less than $\bar{q}_L$ and banks risk to be intervened with probability $e$, while in the latter case $\hat{q} > \bar{q}_L$ and thus these banks will always be allowed to continue operating. Banks with $k \in [\underline{k}_L(e), \bar{k}_L)$ prefer instead not to be intervened and thus choose the supervisory threshold $\bar{q}_L$.

Lemma 2 above characterizes a bank’s behavior for a given anticipated supervisory effort $e$ and level of capital $k$. Note, however, that since the threshold value $\bar{k}_L$ is a function of effort $e$, the point at which a bank switches from its laissez faire choice $\hat{q}$ to the supervisory threshold $\bar{q}_L$ is also a function of $e$. An alternative way of viewing the relationship between bank portfolio choice and supervisory effort is to consider the bank’s
portfolio choice directly as a function of $e$ by solving $D(k) = 0$ for $e$, given a particular level of $k$. Specifically, for $k < \tilde{k}_L$ there exists a level of supervisory effort $\hat{c}(k)$ such that $D(k) = 0$.

Viewed this way, we can now write the bank’s reaction function $\tilde{q}_L(k, e)$ at the individual bank level in the case of a local supervisor as

$$\tilde{q}_L(k, e) = \begin{cases} \tilde{q}(k) & \text{for } e < \hat{c}(k) \text{ and } k < \tilde{k}_L \\ \tilde{q}_L(k) & \text{for } e \geq \hat{c}(k) \text{ and } k < \tilde{k}_L \\ \hat{q}(k) & \text{for } k \geq \tilde{k}_L, \end{cases}$$

where $\tilde{q}(k)$ is as in (1) and $\tilde{q}_L(k)$ is given by (6). Thus, a bank with capital $k > \tilde{k}_L$ is not affected by the threat of regulatory intervention and will always choose its laissez-faire portfolio, while a bank with capital $k < \tilde{k}_L$ will choose its laissez-faire portfolio level only if the supervisor exerts a level of effort below $\hat{c}(k)$, and will meet the supervisory standard $\tilde{q}_L(k)$ for $e > \hat{c}(k)$. Note that, relative to more capitalized banks, less capitalized banks have riskier laissez faire portfolios (lower $\hat{q}$) but are forced into safer portfolios of quality $\tilde{q}_L$ once supervised.

Figure 2 below represents graphically the relationship between bank portfolio choice, $\tilde{q}_L$, and supervisory effort, $e$, for banks with $k < \tilde{k}_L$, for two different levels of capital, $k_1$ and $k_2$. For $e \leq \hat{c}(k_i), i = 1, 2$, $\tilde{q}_L = \tilde{q}$. At $e = \hat{c}(k_i)$, the bank with capital $k_i$ switches to the higher quality portfolio $\tilde{q}_L$. Since $k_1 < k_2$, we also have that $\tilde{q}_L(k_1) > \tilde{q}_L(k_2)$ reflecting our argument above that the supervisor is more lenient with better capitalized banks.

Having derived the bank portfolio choice as a function of the supervisory effort $e$, we can now turn to study the supervisor’s optimal effort choice as a function of the distribution of capital across banks in the economy. Before doing this, we note that although we have derived it as an individual bank’s choice, the threshold $\bar{k}_L(e)$ described in Lemma 2 can be seen as characterizing the set of banks that will be intervened if discovered by the supervisor (those with $k < \bar{k}_L(e)$) as well as those whose behavior/portfolio choice will be affected by supervision (those with $k \in [\bar{k}_L(e), \bar{k}_L]$). In this sense, the threshold $\bar{k}_L(e)$ can be viewed as an aggregate reaction function to the supervisory effort $e$, with $\frac{d\bar{k}_L(e)}{de} < 0$, as argued above. In other words, the banking system’s reaction function for the critical value $\bar{k}_L$ is downward sloping in $e$. 

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4.3 Inspection effort of the local supervisor

We can now turn to the choice of the local regulator concerning the effort \( e \) to exert to inspect the bank. Given the decision to intervene for \( q < \tilde{q}_L(k) \), the supervisor’s objective function is

\[
\max_e SW_L = \Pr (q > \tilde{q}_L) E[q \left( R - \frac{1}{2} cq - (1 - k) \right) - (1 - q) (1 - k) \psi_L - k|q > \tilde{q}_L] \\
+ e \Pr (q < \tilde{q}_L) E[-A_L + q^*_L \left( R - \frac{1}{2} cq - (1 - k) \right) - (1 - q^*_L) (1 - k) \psi_L - k|q < \tilde{q}_L] \\
+(1 - e) \Pr (q < \tilde{q}_L) E[q \left( R - \frac{1}{2} cq - (1 - k) \right) - (1 - q) (1 - k) \psi_L - k|q < \tilde{q}_L] - \frac{e^2}{2},
\]

(13)

where \( \tilde{q}_L < q^*_L \) is the intervention threshold as given in (6) and \( q^*_L \) is the bank portfolio the supervisor implements upon intervention, as given by (4). The first term represents the expected social surplus when the bank chooses an investment portfolio of quality \( q > \tilde{q}_L \) and is allowed to continue. The second term represents the case when the supervisor inspects the bank and intervenes, while the third term is the payoff when the supervisor is unsuccessful in its inspection and the bank can continue its investment project despite having chosen a portfolio with \( q < \tilde{q}_L \). Finally, the last term represents the cost of inspection.

For simplicity we can rewrite the supervisor’s objective function as

\[
SW_L = E[q \left( R - \frac{1}{2} cq - (1 - k) \right) - (1 - q) (1 - k) \psi_L - k] + e \Pr (q < \tilde{q}_L) E[I_L(q^*_L, q, k)|q < \tilde{q}_L] - \frac{e^2}{2},
\]

(14)
where \( I_L(q^*_L, q, k) \) is the net gain from intervention for the supervisor as defined in (5). Clearly, only the last two terms depend on the supervisor’s inspection effort. This means that the supervisor will choose the inspection effort taking into account that this will affect only banks with portfolios of quality \( q < \bar{q}_L \) or, equivalently, those with \( k < k_L(e) \) as defined in Lemma 2. We can then derive the supervisor’s optimal effort as follows.

**Lemma 3** Given the bank’s optimal behavior as described in Lemma 2, the local supervisor chooses a level of effort \( e_L(\bar{k}_L) \) given by

\[
e_L(\bar{k}_L) = \min \left\{ \int_{0}^{\bar{k}_L} I_L(q^*_L, \hat{q}, k) f(k) \, dk, \bar{e}_L \right\}.
\]

(15)

It follows that \( \frac{\partial e_L(\bar{k}_L)}{\partial k_L} > 0 \).

**Proof.** Differentiating (14) with respect to \( e \) and recalling we have restricted \( e \) to be less than \( \bar{e}_L \) as in (12) gives

\[
e_L = \min \left\{ \Pr (\hat{q} < \bar{q}_L) E[I_L(q^*_L, \hat{q}, k)|\hat{q} < \bar{q}_L], \bar{e}_L \right\}.
\]

(16)

Further, from Lemma 2, it follows that in equilibrium \( \Pr (\hat{q} < \bar{q}_L) = \Pr (k < \bar{k}_L(e)) = F(\bar{k}_L) \) and \( E[I_L(q^*_L, \hat{q}, k)|\hat{q} < \bar{q}_L] = \frac{1}{F(\bar{k}_L)} \int_{0}^{\bar{k}_L} I_L(q^*_L, \hat{q}, k) f(k) \, dk \). Substituting these into the expression above for \( e_L \) gives (15). Differentiating this with respect to \( \bar{k}_L \) gives

\[
\frac{\partial e_L}{\partial \bar{k}_L} = I_L(q^*_L, \hat{q}(\bar{k}_L), \bar{k}_L) f(\bar{k}_L) > 0.
\]

The lemma describes the supervisor’s reaction function in the case of local supervision. The supervisory effort depends positively on the threshold \( \bar{k}_L \) characterizing banks’ optimal behavior. The higher \( \bar{k}_L \) and thus the greater is the fraction of banks with \( k < \bar{k}_L \) that will choose \( \hat{q} < \bar{q}_L \) and thus the higher the supervisory effort.

### 4.4 Equilibrium

Having characterized the reaction functions for the supervisor and the banks, we can now characterize the equilibrium.

**Proposition 4** For given intervention cost \( A_L \) and shadow cost of public funds \( \psi_L \), there is a unique equilibrium \( (e^*_L, \bar{q}_L, \hat{q}_L) \) in supervisory effort, intervention strategy, and bank portfolio choices such that:
1) The supervisor will intervene (upon obtaining actionable information) any bank with \( q < \tilde{q}_L(k) \), where \( \tilde{q}_L(k) \) is as in (6);

2) Banks that choose not to meet the supervisor’s standards, \( \tilde{q}_L(k) \), choose their laissez-faire optimal portfolios \( \tilde{q}(k) \);

3) The optimal supervisory effort \( \epsilon^*_L \) and capital threshold \( \tilde{k}_L^* \) below which banks choose not to meet supervisory standards satisfy: \( \epsilon_L \left( \tilde{k}_L^* \right) = \epsilon^*_L \) and \( \tilde{k}_L \left( \epsilon^*_L \right) = \tilde{k}_L^* \).

Proof. To establish the existence and uniqueness of an equilibrium, recall first that we can focus on the equilibrium as a function of the supervisory effort \( \epsilon \) and capital threshold level \( \tilde{k}_L \), rather than portfolio choice \( q(k) \), since, as described in Lemma 2, \( \tilde{k}_L \) provides a summary statistic of how supervisory effort affects bank risk taking behavior. Furthermore, from the lemmas above we have \( \frac{\partial \tilde{k}_L}{\partial \epsilon} < 0 \) and \( \frac{\partial \epsilon^*_L}{\partial \tilde{k}_L} > 0 \), so that the two reaction functions will intersect at most once.

For \( \epsilon = 0 \), the function \( \tilde{k}_L(e) \) equals the regulatory threshold \( \tilde{k}_L \), since for \( \epsilon = 0 \) only banks with \( k \geq \tilde{k}_L \) will choose \( q > \tilde{q}_L \). Conversely, \( \tilde{k}_L(e) \) will equal 0 at \( \tilde{\epsilon}_L \) as defined in (12). It follows that the two functions can only cross once and that \( \tilde{k}_L^* \in \left( 0, \tilde{k}_L \right) \) and \( \epsilon^*_L \in \left( 0, \tilde{\epsilon}_L \right) \). To show that the solution must be strictly interior, note that \( \epsilon^*_L = \tilde{\epsilon}_L \) cannot be an equilibrium as then \( \tilde{k}_L(\tilde{\epsilon}_L) = 0 \) and all banks would choose \( q \geq \tilde{q}_L \).

Given this, it cannot be optimal for the supervisor to choose \( \epsilon^*_L = \tilde{\epsilon}_L \) since all banks are meeting the regulatory standards. Likewise, \( \tilde{k}_L = 0 \) cannot be an equilibrium as then the supervisor’s optimal response would be to choose \( \epsilon = 0 \), which would make the proposed solution of \( \tilde{k}_L = 0 \) not rational for the banks. Therefore, the equilibrium must be strictly interior. □

Proposition 4 establishes that a unique equilibrium exists where the supervisor exerts a strictly positive level of effort in identifying which banks may have portfolios that it views as excessively risky, and some banks adjust their behavior to conform to the supervisory standards. Figure 3 illustrates the result. The downward sloping line represents the banks’ reaction function, \( \tilde{k}_L(e) \), as a function of supervisory effort. The upward sloping line represents the supervisor’s reaction function \( \epsilon_L \left( \tilde{k} \right) \) as a function of the threshold level of capital below which banks choose their laissez-faire portfolios. The point where they intersect is the equilibrium, and implies a strictly positive level of effort, \( \epsilon^*_L \), as well as a threshold level of capital \( \tilde{k}_L^* \) strictly less than \( \tilde{k}_L \), meaning that supervision leads a strictly positive measure of banks to move away from their most preferred portfolio \( \tilde{q} \) and instead adhere to the regulatory standard \( \tilde{q}_L \).
Figure 3: Equilibrium with local supervision. The figure describes the bank’s reaction function $k_L(e)$ and the local supervisor’s reaction function $e_L(k_L)$ as functions of supervisory effort $e$ and capital threshold $k_L$, respectively.

5 The centralization of bank supervision

So far, we have considered the case where there is only a single, local supervisor, and studied the implications of that supervisor’s intervention decision on the bank’s choice of portfolio risk. Here, we introduce a central supervisor who has the power to mandate intervention, but who must rely on the local supervisor to obtain actionable information before it can intervene.

Consider therefore the following extension to the model. A central supervisor has interest in maintaining a healthy banking sector and minimizing the need for costly intervention, much as the local regulator, but faces somewhat different tradeoffs. In particular, the central supervisor perceives the cost of intervention as $A_C \neq A_L$ and/or the shadow cost of funds as $\psi_C \neq \psi_L$, so that it is more/less willing to intervene than the local regulator. This difference in the perceived costs may represent the central supervisor’s internalization of bank failures on the overall financially integrated area, or may simply reflect that the local supervisor is partially “captured” by local constituents, including banks, while the central supervisor is less likely to attach much weight to local political economy considerations. Regardless, the implication is that the central supervisor may mandate intervention by the local supervisor in situations where the local supervisor would
prefer to forbear and allow the bank to operate unimpeded, or vice versa. The rest of the model remains unchanged.

We can now study the central supervisor’s intervention decision. As in Section 4.1, if the local supervisor successfully obtained information about a given bank’s portfolio, the central supervisor must decide whether it is optimal to intervene at that bank or not. The central supervisor compares intervening, which entails the cost $A_C$ but gives a payoff

$$-A_C + q_C^* \left( R - \frac{1}{2} cq_C^* - (1 - k) \right) - (1 - q_C^*) (1 - k) \psi_C - k,$$

where

$$q_C^* = \frac{R + (1 - k) (\psi_C - 1)}{c}$$

is the central supervisor’s preferred bank portfolio conditional on intervention, and forbearing, which allows the bank to continue with its initial portfolio choice $q$ and gives the supervisor a payoff equal to

$$q \left( R - \frac{1}{2} cq - (1 - k) \right) - (1 - q) (1 - k) \psi_C - k.$$

It is straightforward to see that the comparison is the same as that in (5), only with a different intervention cost $A_C$ and shadow cost of funds $\psi_C$. Therefore, following the analysis in Section 4.1, we can define

$$I_C(q_C^*, q, k) = -A_C + q_C^* \left( R - \frac{1}{2} cq_C^* - (1 - k) \right) - (1 - q_C^*) (1 - k) \psi_L$$

$$- \left( q \left( R - \frac{1}{2} cq - (1 - k) \right) - (1 - q) (1 - k) \psi_L \right),$$

and find the optimal intervention threshold for the central supervisor below which intervention will occur as given by

$$\tilde{q}_C(k) = \frac{1}{c} \left( R + (1 - k) (\psi_C - 1) - \sqrt{2cA_C} \right).$$

It is worth noting that, as for the case of independent supervision, $\tilde{q}_C(k)$ represents a subgame perfect solution when the central supervisor cannot precommit to an intervention threshold involving a time inconsistency. Moreover, as in the previous case we require $\tilde{q}_C(k)$ to be positive at least for some values of $k$, for which assuming that $\psi_C > \sqrt{2cA_C}$ is sufficient.

The relative position of the central, $\tilde{q}_C(k)$, and local, $\tilde{q}_L(k)$, intervention thresholds (and their slope with respect to $k$) will depend on the relative magnitude of the parameters that determine the cost of intervention
and eventual deposit insurance outlays: $A_C, A_L, \psi_C$, and $\psi_L$. Any $\tilde{q}_C \neq \tilde{q}_L$ may entail inefficient information collection by the local regulator, but the nature of the inefficiency may be different depending on whether $\tilde{q}_C > \tilde{q}_L$ or $\tilde{q}_C < \tilde{q}_L$, that is on whether the central supervisor is tougher or more lenient than the local one. Looking at the expressions given in (6) and (18) gives the following immediate result.

**Lemma 5** (1) Assume that $\psi_C = \psi_L$, so that both supervisors face the same shadow cost of funds. Then, the central supervisor is tougher in his intervention policy than the local supervisor if he faces a lower intervention cost, i.e.: $\tilde{q}_C (k) > \tilde{q}_L (k)$ if $A_C < A_L$, and is more lenient otherwise. (2) Assume that $A_C = A_L$, so that both supervisors bear the same cost of intervention. Then, the central supervisor is tougher if he faces a higher shadow cost of funds, i.e., $\tilde{q}_C (k) > \tilde{q}_L (k)$ if $\psi_C > \psi_L$, and is more lenient otherwise.

Note that the case where $\psi_C > \psi_L$ implies also that the central supervisor will choose, upon intervention, a higher portfolio quality for the bank than the local supervisor, i.e., $q^*_C > q^*_L$. The intuition is simple. If the central supervisor faces a higher shadow cost of repaying depositors when the bank fails, or internalizes externalities associated with a bank’s failure, he will want to prevent bank failure more than the local supervisor and will therefore choose to implement a higher portfolio quality upon intervention.

In what follows, we focus on the case where the central supervisor is tougher and we analyze the implications this has for bank risk taking and supervisory effort in equilibrium.

### 5.1 An unambiguously tougher central supervisor

Given a tougher intervention policy of the central regulator, $\tilde{q}_C (k) > \tilde{q}_L (k)$, either because of a lower intervention cost $A_C < A_L$ or because of a higher shadow cost of funds, $\psi_C > \psi_L$, we can now turn to the bank’s portfolio decision choice and the supervisory effort, which is undertaken by the local supervisor. The analysis follows similar steps to those in the case of the local supervisor.

To see how the threat of regulatory intervention affects banks’ choice of portfolio risk, we first find the threshold $\tilde{k}_C$ at which banks independently would choose a portfolio $\tilde{q}(k)$ which is sufficiently safe not to be intervened: $\tilde{q}(k) = \tilde{q}_C(k)$. Equating (1) and (18) and solving for $k$ gives

$$\tilde{k}_C = \left(1 - \frac{1}{\psi_C} \sqrt{2cA_C} \right).$$

Again, we assume that $\tilde{k}_C > 0$, which is true if $\psi_C > \sqrt{2cA_C}$. Note also that $\tilde{k}_C > \tilde{k}_L$ if either $\psi_C > \psi_L$ or $A_C < A_L$, meaning that, when the central regulator is tougher, banks are less likely to choose voluntarily a
portfolio sufficiently safe to satisfy the supervisor, i.e., that has a quality $q$ at least equal to the supervisory intervention threshold $q_C(k)$.

We can now see how the threat of intervention affects the choice of banks with capital less than the threshold $k_C$. Following the same analysis as in Section 4.2, we obtain the following.

**Lemma 6** For a given level of supervisory effort $e$, there exists a level of capital $k_C(e)$ such that banks with $k < k_C(e)$ choose the laissez-faire portfolio quality $q(k)$, while banks with $k \in [k_C(e), \tilde{k}_C]$ switch to a portfolio of quality $q_C(k) > q(k)$, where

$$k_C(e) = \left(1 - \frac{R\sqrt{e} + \sqrt{2cA_C}}{\psi_C + \sqrt{e}}\right) < \tilde{k}_C.$$  \hspace{1cm} (20)

It follows that $\frac{\partial k_C(e)}{\partial e} < 0$, $\frac{\partial k_C(e)}{\partial \psi_C} > 0$ and $\frac{\partial k_C(e)}{\partial A_C} < 0$.

**Proof.** The proof follows the same steps as the proof of Lemma 2. A bank with $k < k_C$ will choose to switch from a portfolio of quality $q$ to one of quality $q_C(k)$ if it has higher expected profit from doing so. The relevant expressions to compare are (10) using $q_C(k)$ in place of $q_L(k)$ and (9). Substituting the expression for $q_L(k)$ as in (18) into (10) and $q$ as in (1) into (9), setting the difference equal to zero, and solving for $k$ gives $k_C(e)$ as in (20). Comparing this to $k_C$ as in (19) gives $k_C(e) < k_C$. Moreover, it follows easily $\frac{\partial k_C(e)}{\partial e} = -\frac{\psi_C R - \sqrt{2cA_C}}{2\sqrt{e(\psi_C + \sqrt{e})^2}} < 0$, $\frac{\partial k_C(e)}{\partial \psi_C} = \frac{R\sqrt{e} + \sqrt{2cA_C}}{(\psi_C + \sqrt{e})^2} > 0$ and $\frac{\partial k_C(e)}{\partial A_C} = -\frac{e}{\sqrt{2cA_C(\psi_C + \sqrt{e})}} < 0$. The lemma follows. \hfill $\blacksquare$

The lemma identifies the banks whose portfolio choice is affected by the threat of intervention when a central supervisor is present. As in Section 4.2, the function $k_C(e)$ can in fact be seen both as an individual bank’s reaction function for given supervisory effort $e$ and as an aggregate reaction function characterizing the proportion of banks with $k < k_C$ that will be intervened if discovered and those with $k \in [k_C(e), \tilde{k}_C]$ whose portfolio choice is affected by supervision.

As before, for the supervisory intervention to be meaningful in the model, we assume that $k_C(e) > 0$. Setting (20) equal to zero and solving for $e$, this means requiring the parameters $R, \psi_C$ and $A_C$ are such that in equilibrium the local supervisor will choose an inspection level $e < \tilde{e}_C$, where

$$\tilde{e}_C = \frac{(\psi_C - \sqrt{2cA_C})^2}{(R - 1)^2}.$$  \hspace{1cm} (21)

Comparing the expressions for $k_C(e)$ and $k_L(e)$ as in (20) and (11), we obtain the following result.
Lemma 7 If the central supervisor is tougher than the local supervisor, i.e., if $A_C < A_L$ for $\psi_C = \psi_L$, or if $\psi_C > \psi_L$ for $A_C = A_L$, so that $\hat{q}_C(k) > \hat{q}_L(k)$, then the bank’s reaction function $\tilde{k}_C(e)$ lies above the reaction function under independence: $\tilde{k}_C(e) > \tilde{k}_L(e)$ for all $e$.

The lemma establishes that, for a given supervisory effort $e$, $\tilde{k}_C(e) > \tilde{k}_L(e)$, meaning that there will be banks with $k \in (\tilde{k}_L(e), \tilde{k}_C(e)]$ that will no longer comply with the minimum supervisory portfolio quality under central supervision and will therefore risk being intervened. Interestingly, this does not necessarily imply that fewer banks comply with regulatory standards under central supervision since the threshold $\tilde{k}_C$ below which banks would choose the laissez-faire portfolio quality in the absence of a supervisory intervention threat is now also higher. To see when the supervisory threat is has a greater effect on banks’ behavior, we compare $\tilde{k}_C - \tilde{k}_C(e)$ with $\tilde{k}_L - \tilde{k}_L(e)$, the range of banks who deviate from their preferred portfolios to avoid being intervened under centralization and with only a local supervisor, respectively. Substituting all the relevant expressions, we have

$$\tilde{k}_C - \tilde{k}_C(e) = \frac{\sqrt{e}(R\psi_C - \sqrt{2\varepsilon A_C})}{\psi_C(\psi_C + \sqrt{e})}$$

and

$$\tilde{k}_L - \tilde{k}_L(e) = \frac{\sqrt{e}(R\psi_L - \sqrt{2\varepsilon A_C})}{\psi_L(\psi_L + \sqrt{e})}.$$ 

For the case where both supervisors have the same shadow cost, $\psi_C = \psi_L$, but the central supervisor has a lower cost of intervention, $A_C < A_L$, the expressions above show that $\tilde{k}_C - \tilde{k}_C(e) > \tilde{k}_L - \tilde{k}_L(e)$, so that there is a larger range of banks for which the threat of supervisory intervention leads them to adjust their portfolio upward when there is a central supervisor compared to when there is only a local supervisor.

Given the bank’s portfolio choice for a given level of supervisory effort $e$, we can now study the local supervisor’s effort problem when the central supervisor decides whether to intervene or not, but must rely on the information collected by the local supervisor in order to act. In the case when he was acting independently, the local supervisor’s effort problem was given by (14). Essentially, the intervention decision of the local supervisor, $\hat{q}_L$, partitioned the mass of banks into two regions, and effort was a function of the mass of banks with $q < \hat{q}_L$ and the average benefit from intervention in that region, $E[I_L(q^*_L, q, k) | q < \hat{q}_L]$. Here, however, since $\hat{q}_C > \hat{q}_L$ so that the two supervisors have different intervention thresholds, there may be a region where the supervisor’s effort is in fact decreasing in the set of banks that are subject to being
intervened. Moreover, to the extent that the portfolio quality that will be implemented upon intervention, \( q^*_C \), may be different under centralization than that under independence, \( q^*_L \), that will further reduce the local supervisor’s incentives to exert effort, since intervention can only occur when information is collected.

When the central supervisor is tougher than the local one, banks will be partitioned into three groups. For \( q \geq \tilde{q}_C \), learning the bank’s type \((q, k)\) brings no benefit to the local supervisor because those banks are not intervened. For \( q < \tilde{q}_L \), banks are intervened as under independent local supervision. However, if \( q^*_C \neq q^*_L \) the local supervisor’s benefits are not the same as before, and are in fact lower since \( q^*_C \) instead of \( q^*_L \) will be implemented. This will imply a loss for the local supervisor relative to the case of independence, implying that even for banks that would otherwise choose \( \tilde{q} \) close to \( \tilde{q}_L \), the local supervisor will incur a loss (recall that \( \tilde{q} = \tilde{q}_L \) is the point where the local supervisor is exactly indifferent between intervening and not when intervention calls for choosing the portfolio \( q^*_L \)). We define the point at which the local supervisor is indifferent between intervention and not under central supervision as \( \tilde{q}(q^*_C) \), and note that \( \tilde{q}(q^*_C) = \tilde{q}_L \) if \( q^*_C = q^*_L \). In other words, for \( \tilde{q}(q^*_C) \leq q < \tilde{q}_C \), learning the bank’s type entails a loss equal to \( I_L(q^*_C, q, k) < 0 \) for the local regulator. This is because he would rather let these banks continue than intervene them and instead is forced to intervene.

The argument above implies that the local supervisor’s problem now becomes

\[
\max_{e} SW^C_L = \Pr(q > \tilde{q}_C) E[q \left( R - \frac{1}{2}cq - (1-k) \right) - (1-q)(1-k)\psi_L - k|q > \tilde{q}_C] \\
+ \epsilon \Pr(q < \tilde{q}(q^*_C)) [ -A_L + q^*_C \left( R - \frac{1}{2}cq^*_C - (1-k) \right) - (1-q^*_C)(1-k)\psi_L - k|q < \tilde{q}(q^*_C) ] \\
+ \epsilon \Pr(\tilde{q}(q^*_C) \leq q < \tilde{q}_C) [ -A_L + q^*_C \left( R - \frac{1}{2}cq^*_C - (1-k) \right) - (1-q^*_C)(1-k)\psi_L - k|\tilde{q}(q^*_C) \leq q < \tilde{q}_C] \\
(1-\epsilon) \Pr(q < \tilde{q}(q^*_C)) E[q \left( R - \frac{1}{2}cq - (1-k) \right) - (1-q)(1-k)\psi_L - k|q < \tilde{q}(q^*_C) ] \\
+(1-\epsilon) \Pr(\tilde{q}(q^*_C) \leq q < \tilde{q}_C) E[q \left( R - \frac{1}{2}cq - (1-e) \right) - (1-q)(1-k)\psi_L - k|\tilde{q}(q^*_C) \leq q < \tilde{q}_C] - \frac{\epsilon^2}{2}.
\]

This reduces to

\[
\max_{e} SW^C_L = E[q \left( R - \frac{1}{2}cq - (1-k) \right) - (1-q)(1-k)k] \\
+ \epsilon \Pr(q < \tilde{q}(q^*_C)) [ I_L(q^*_C, q, k)|q < \tilde{q}(q^*_C) ] \\
+ \epsilon \Pr(\tilde{q}_L \leq q < \tilde{q}(q^*_C)) E[I_L(q^*_C, q, k)|\tilde{q}(q^*_C) \leq q < \tilde{q}_C] - \frac{\epsilon^2}{2}.
\]
Given that \( E[I_L(q^*_C, q, k)|q < \bar{q}(q^*_C)] < E[I_L(q^*_L, q, k)|q < \bar{q}(q^*_C)] \) if \( q^*_C \neq q^*_L \) and \( E[I_L(q^*_C, q, k)|\bar{q}(q^*_C) \leq q < \bar{q}_C] < 0 \), the expression for \( SW^C_L \) above is smaller than the expression for \( SW^C_L \) in (14). Analogously to above, we define \( \tilde{k}(q^*_C) \) as the value of \( k \) for which \( \bar{q}(k) = \bar{q}(q^*_C) \). Assuming that the banks behave as above, choosing either \( \bar{q} \) or \( \bar{q}_C \), we obtain the following.

**Lemma 8** If the central regulator is tougher than the local regulator, that is if \( A_C < A_L \) for \( \psi_C = \psi_L \), or if \( \psi_C \) > \( \psi_L \) for \( A_C = A_L \) so that \( \bar{q}_C(k) > \bar{q}_L(k) \) and \( q^*_C \geq q^*_L \), the local supervisor’s reaction function, as given by

\[
e_{C}(\bar{k}_C) = \min \left\{ \int_0^{\min(\bar{k}_C, \tilde{k}(q^*_C))} I_L(q^*_C, \bar{q}, k) f(k) \, dk + 1_{\bar{k}_C > \tilde{k}(q^*_C)} \int_{\tilde{k}(q^*_C)}^{\bar{k}_C} I_L(q^*_C, \bar{q}, k) f(k) \, dk, \bar{r}_C \right\}, \tag{23}
\]

lies (weakly) below the reaction function under independence: \( e_{C}(\bar{k}) \leq e_{L}(\bar{k}) \).

The lemma characterizes the local supervisor’s reaction function under central supervision. As it shows, we can decompose the reaction function as

\[
e_{C} = \begin{cases} 
\int_0^{\bar{k}(q^*_C)} I_L(q^*_C, \bar{q}, k) f(k) \, dk & \text{for } \bar{k}_C \leq \bar{k}(q^*_C) \\
\int_{\bar{k}(q^*_C)}^{\bar{k}_C} I_L(q^*_C, \bar{q}, k) f(k) \, dk + \int_{\bar{k}(q^*_C)}^{\bar{k}_C} I_L(q^*_C, \bar{q}, k) f(k) \, dk & \text{for } \bar{k}_C > \bar{k}(q^*_C),
\end{cases} \tag{24}
\]

where \( I_L(q^*_C, \bar{q}, k) f(k) \, dk > 0 \) for \( \bar{k}_C \leq \bar{k}(q^*_C) \) and \( I_L(q^*_C, \bar{q}, k) < 0 \) for \( \bar{k}_C > \bar{k}(q^*_C) \). Given that

\[
\frac{\partial e_{C}}{\partial \bar{k}_C} = \begin{cases} 
I_L(q^*_C, \bar{q}, \bar{k}_C) f(\bar{k}_C) & \text{for } \bar{k}_C \leq \bar{k}(q^*_C) \\
I_L(q^*_C, \bar{q}, \bar{k}_C) f(\bar{k}_C) & \text{for } \bar{k}_C > \bar{k}(q^*_C),
\end{cases}
\tag{25}
\]

it follows that the reaction function \( e_{C}(\bar{k}_C) \) is positively sloped for \( \bar{k}_C \leq \bar{k}(q^*_C) \) and negatively sloped for \( \bar{k}_C > \bar{k}(q^*_C) \).

The lemma establishes that, for a given \( \bar{k}_C \), when supervisory powers are centralized, the local supervisor will be, all things equal, (weakly) less diligent in trying to uncover excessive risk taking on the side of the banks under its jurisdiction. This occurs for the simple reason that once any hard information is uncovered, the central supervisor can use that to intervene if the bank is discovered to have chosen a riskier portfolio than what the supervisor desires. In some of these instances, the local supervisor would prefer not to intervene because it faces a larger intervention cost relative to the benefit it perceives from adjusting the bank’s portfolio, but does not have the authority to make this decision. As a consequence, the local supervisor instead chooses to reduce the likelihood that any damning evidence is found, meaning that it reduces its effort relative to what it would find individually optimal to do.

Having characterized the reaction function \( \bar{k}_C(e) \) of the banks and that of the supervisor \( e_{C}(\bar{k}_C) \), we can now turn to characterize the equilibrium. To do this, we distinguish the analysis into two cases: the case
where the central regulator has lower intervention cost than the local regulator \((A_C < A_L)\) but equal shadow cost \((\psi_C = \psi_L)\), and the case where the central regulator has higher shadow cost than the local regulator \((\psi_C > \psi_L)\) but it has the same intervention cost \((A_C = A_L)\). In both cases the central regulator is tougher in that it intervenes at a higher level of portfolio quality, \(\bar{q}_C > \bar{q}_L\), than the local regulator. However, in the former case \(q^*_C = q^*_L\) so that the supervisory portfolio quality does not vary with centralized supervision, while in the latter \(q^*_C > q^*_L\). This affects the equilibrium values \((e^*_C, \bar{k}_C)\) differently as we will see below.

### 5.2 Lower intervention cost for the central regulator: \(A_C < A_L\) and \(\psi_C = \psi_L\)

We start with the case where \(A_C < A_L\) and \(\psi_C = \psi_L\), so that the central supervisor has a lower intervention threshold and thus is tougher, but since he has the same shadow cost of funds he chooses the same portfolio conditional on intervention as the local supervisor. Using the notation from the model, this means \(\bar{q}_C > \bar{q}_L\) but \(q^*_C = q^*_L\). It follows from Lemmas 7 and 8 that the local supervisor’s reaction function under centralization coincides with the one under local supervision (15) for \(\bar{k}_C \leq \bar{k}_L\), while the bank’s reaction function \(\bar{k}_C(e)\) under centralization remains strictly above the one under local supervision, \(\bar{k}_L(e)\).

We can now turn to analyze the equilibrium levels of the supervisory effort and banks’ choice of portfolio risk, as well as the equilibrium risk level of the banking sector’s portfolio. In what follows we restrict our attention the case where capital is uniformly distributed across the banks, i.e., \(k \sim U[0,1]\). We have the following result.

**Proposition 9** Suppose that \(\psi_C = \psi_L\). There is a value \(\delta > 0\) such that, for \(A_L - A_C < \delta\), the local supervisor exerts more effort under centralization than when independent, \(e^*_C > e^*_L\), and banks’ average portfolio quality is higher: \(\int_0^1 q_C(k) f(k) dk > \int_0^1 q_L(k) f(k) dk\), where \(q_C(k)\) and \(q_L(k)\) are banks’ equilibrium choices of portfolio quality under central and local supervision, respectively.

The proposition establishes that when the conflict between the local and the central supervisor is relatively small, meaning that their respective costs of intervention are not very different, centralizing bank supervision leads to an increase in the effort exerted by the local supervisor in detecting banks’ portfolio choices, and consequently also leads to safer portfolios. The reason is simply that the greater toughness of the central supervisor forces banks to choose safer portfolios to avoid being intervened. The local supervisor, while not always fully in agreement with the central supervisor, exerts more effort in order to try to catch the banks that would have behaved in the absence of a central supervisor, but now find it too onerous to do so given
Figure 4: Equilibrium with central supervision in the case of a tougher central regulator. The figure describes the bank’s reaction function $k_C(e)$ and the local supervisor’s reaction function $e_C(k_C)$ as a function of supervisory effort $e$ and bank capital threshold $k_C$, respectively.

the tougher standard.

However, centralization of supervision creates an agency problem vis-à-vis the local supervisor who must collect actionable information about the banks’ operations. As the following result shows, this may lead to less diligence on the part of the local supervisor when the conflict with the central supervisor is sufficiently large.

**Proposition 10** Suppose that $\psi_C = \psi_L$, and that $A_C = 0$ so that the central supervisor faces no cost of intervention. Then, there is a value $\bar{A}$ such that, for $A_L > \bar{A}$, supervisory effort decreases under centralization: $e_C^* < e_L^*$.

**Proof.** To be completed. □

The intuition for the last result is that, when the divergence between the objectives of the central and the local supervisors is sufficiently large, the local supervisor’s incentives to exert effort are sufficiently muted that less information collection takes place in equilibrium. This results in a weaker intervention threat for banks, since there it is less likely that actionable information will be obtained. If the conflict between the supervisors is sufficiently big, overall portfolio quality may go down despite the tougher regulatory standards.
5.3 Higher shadow cost for the central regulator: \( A_C = A_L \) and \( \psi_C > \psi_L \)

We now investigate the case where the central supervisor faces the same intervention cost as the local supervisor \( A_C = A_L \), but has a higher shadow cost \( \psi_C > \psi_L \). This implies again \( \bar{q}_C > \bar{q}_L \), but also that \( q^*_C > q^*_L \) so that the supervisor is now tougher in terms of both having a higher intervention threshold and adopting a higher standard conditional on intervention. It follows then from Lemma 7 that the local supervisor’s reaction function under centralization lies strictly below the one under local supervision (15) for \( \bar{k}_C \leq \bar{k}_L \) because \( I_L (q^*_C, \bar{q}, k) < I_C (q^*_L, \bar{q}, k) \) for any \( \bar{k}_C \leq \bar{k}_L \), while, as stated in Lemma 7 above, the bank’s reaction function \( \bar{r}_C (e) \) under centralization remains strictly above the one under local supervision \( \bar{r}_L (e) \).

**Lemma 11** When \( A_C = A_L \) and \( \psi_C > \psi_L \), the local supervisor’s reaction function under centralization is strictly below the reaction function under independence: \( e_C (\bar{k}) < e_L (\bar{k}) \) for all \( \bar{k} < \bar{k}_C \).

**Proof.** Recall that the reaction function under centralization is given by

\[
e_C (\bar{k}_C) = \min \left\{ \int_0^{\min(\bar{k}_C, \bar{k}(q^*_C))} I_L (q^*_C, \bar{q}, k) f (k) dk + \mathbf{1}_{\bar{k}_C > \bar{k}} (q^*_C) \int_{\bar{k}}^{\bar{k}_C} I_L (q^*_C, \bar{q}, k) f (k) dk, \bar{k}_C \right\},
\]

where \( \bar{k} (q^*_C) \leq \bar{k}_L \). Given that \( E[I_L (q^*_C, \bar{q}, k) | \bar{q} < \bar{q} (q^*_C)] < E[I_L (q^*_L, \bar{q}, k) | \bar{q} < \bar{q} (q^*_L)] \) if \( q^*_C \neq q^*_L \), this establishes that \( e_C (\bar{k}) < e_L (\bar{k}) \) for \( \bar{k} < \bar{k} (q^*_C) < \bar{k}_C \). For \( \bar{k} > \bar{k} (q^*_C) \), we have that \( E[I_L (q^*_C, \bar{q}, k) | \bar{q} (q^*_C) \leq \bar{q} < \bar{q}_C] < 0 \), so the result trivially follows also for \( \bar{k} > \bar{k} (q^*_C) \).

The lemma, which is a special case of Lemma 8, establishes that when the central supervisor is tougher because he has a higher shadow cost of funds, the local supervisor will react by, ceteris paribus, exerting less effort. The reason is simply that the difference in the shadow costs translates into a difference in the portfolios that each supervisor would like to see implemented conditional on intervention, \( q^*_C \) and \( q^*_L \), so that even in cases where the local supervisor would like to intervene (because the bank has chosen an excessively risky portfolio), he does not get to implement his preferred portfolio. This conflict between the two supervisors reduces the incentives of the local supervisor to exert effort at identifying high-risk banks.

Figure 5 illustrates the result from Lemma 11. The red curve labeled \( e_C \) represents the local supervisor’s reaction function for effort when the central supervisor has a higher shadow cost of funds. In particular, \( e_C \) is strictly below \( e_L \) for all levels of the capital threshold \( \bar{k} \), reflecting that, ceteris paribus, the local supervisor prefers to put in less effort than he would under independence. The curve labeled \( \bar{k}_C \), however, represents the banks’ reaction to the tougher standards, and is shifted out relative to the case of an independent local supervisor, much as in Section 5.2. Now, however, the implications of having a tougher central supervisor
Figure 5: Equilibrium with central supervision when supervisors have different shadow costs of funds. The figure describes the bank’s reaction function $\tilde{k}_C(e)$ and the local supervisor’s reaction function $e_C(\tilde{k}_C)$ as a function of supervisory effort $e$ and bank capital threshold $\tilde{k}_C$, respectively.

are more ambiguous: while banks respond by themselves adopting higher quality portfolios for a given level of effort, the supervisor responds by exerting less effort. In principle, this can lead to either more or less effort being exerted in equilibrium.

5.4 Numerical examples

To be completed.

6 Conclusion

This paper develops a simple, static model to analyze the effects of a bank supervisory system where local supervisors are tasked with the day to day job of monitoring banks and identifying which have risky portfolios and are poorly capitalized, but where the consequence associated with identifying such banks are under the control of a central supervisor who may have different objectives. We show first that the threat of regulatory intervention provides a disciplining force for banks, and causes some banks to adjust their investment decisions, choosing safer portfolios than they otherwise would. Banks who find it very costly to
comply with regulatory standards due to being too poorly capitalized will instead stick with their preferred portfolios, but will risk being intervened and losing the returns from their investments.

With a central supervisor, the incentives to collect information decrease for the local supervisor whose task is to identify high risk bank and implement the intervention policies of the central supervisor. This agency conflict can lead to less information being collected and, as a result, worse quality portfolios on aggregate despite the higher regulatory standards.

Our analysis has bearing on the current debate and implementation of the Single Supervisory Mechanism (SSM) in the European Union, where bank supervision is being centralized but local supervisors still represent - and will likely do so for some time - the “boots on the ground,” being the parties that actually have greater contact with local financial institutions and would thus be in the best position to evaluate banks’ portfolios. Our analysis shows that while regulatory standards may increase, and a central supervisor may be put in place to deal with the perceived laxness and unwillingness to intervene that preceded the recent crisis, the outcome may not be to reduce bank risk taking. Rather, to the extent that the parties in charge of information collection and implementation continue to have different objectives, the tougher standards may in fact lead to even greater risk taking and consequently an increased chance of systemic problems down the road.
Appendix

Proof of Lemma 8: Differentiating (22) with respect to \(e\) and recalling we have restricted \(e\) to be less than \(\tau_C\) as given in (21) gives

\[
e_C(\bar{k}_C) = \min\{\Pr(\bar{q} < \bar{q}(q_C^*)) E[I_L(q_C^*, \bar{q}, k) | \bar{q} < \bar{q}(q_C^*)] + \Pr(\bar{q} > q_C^*) E[I_L(q_C^*, \bar{q}, k) | \bar{q} > q_C^*] \leq \bar{q} < \bar{q}_C, \tau_C\}.
\]

This differs from (16) in two respects. First, if \(q_C^* > q_L^*\), as is the case if \(\psi > \psi_L\), then \(E[I_L(q_C^*, \bar{q}, k) | \bar{q} < \bar{q}(q_C^*)] < E[I_L(q_L^*, \bar{q}, k) | \bar{q} < \bar{q}(q_C^*)]\). Second, \(E[I_L(q_C^*, \bar{q}, k) | \bar{q} > q_C^* \leq \bar{q} < \bar{q}_C] < 0\), meaning that relative to (16) there is now an extra negative term in the supervisor’s equation for effort choice. This implies that the reaction function \(e_C(\bar{k}_C)\) lies weakly below \(e_L(\bar{k}_L)\).

To characterize \(e_C(\bar{k}_C)\), recall first that under laissez-faire banks choose \(\bar{q} < \bar{q}_L\) if \(k < \bar{k}_L\) and that under centralization they choose \(\bar{q} < \bar{q}_C\) if \(k < \bar{k}_C\), with \(\bar{q}_C > \bar{q}_L\). We can then distinguish the case where \(\bar{k}_C < \bar{k}_L\) and the case where \(\bar{k}_C > \bar{k}_L\). The same argument applies here replacing \(\bar{k}_L\) and \(\bar{q}_L\) with \(\bar{k}(q_C^*)\) and \(\bar{q}(q_C^*)\) respectively. In the first case,

\[
\Pr(\bar{q} < \bar{q}_L) E[I_L(q_C^*, \bar{q}, k) | \bar{q} < \bar{q}_L] = F(\bar{k}_C) \frac{1}{F(\bar{k}_C)} \int_{0}^{\bar{k}_C} I_L(q_C^*, \bar{q}, k) f(k) dk = \int_{0}^{\bar{k}_C} I_L(q_C^*, \bar{q}, k) f(k) dk
\]

and \(\Pr(\bar{q}_L < q < \bar{q}_C) E[I_L(q_C^*, \bar{q}, k) | \bar{q}_L \leq \bar{q} < \bar{q}_C] = 0\). In the second case, when \(\bar{k}_C > \bar{k}_L\),

\[
\Pr(\bar{q} < \bar{q}(q_C^*)) E[I_L(q_C^*, \bar{q}, k) | \bar{q} < \bar{q}(q_C^*)] = F(\bar{k}(q_C^*)) \frac{1}{F(\bar{k}(q_C^*))} \int_{0}^{\bar{k}(q_C^*)} I_L(q_C^*, \bar{q}, k) f(k) dk = \int_{0}^{\bar{k}(q_C^*)} I_L(q_C^*, \bar{q}, k) f(k) dk
\]

and

\[
\Pr(\bar{q}(q_C^*) < q < \bar{q}_C) E[I_L(q_C^*, \bar{q}, k) | \bar{q}(q_C^*) \leq q < \bar{q}_C] = \left[ F(\bar{k}_C) - F(\bar{k}(q_C^*)) \right] \frac{1}{F(\bar{k}(q_C^*))} \int_{\bar{k}(q_C^*)}^{\bar{k}_C} I_L(q_C^*, \bar{q}, k) f(k) dk
\]

\[
= \int_{\bar{k}(q_C^*)}^{\bar{k}_C} I_L(q_C^*, \bar{q}, k) f(k) dk.
\]

The lemma follows.

Proof of Proposition 9: From Proposition 4, we know that there is a unique equilibrium \((e_L^*, \bar{k}_L^*)\) for the case where there is only a local supervisor. This equilibrium is interior, i.e., \(e_L^* < \tau_L\) and \(\bar{k}_L^* < \bar{k}_L\), with both \(e_L^*\) and \(\bar{k}_L^*\) being strictly positive. This equilibrium also holds for the case of a central supervisor if \(\psi = \psi_L\) and \(A_C = A_L\), so that the central and the local supervisors share the same objectives and there is no conflict between. Moreover, for \(A_C < A_L\) and \(\psi = \psi_L\), \(e_C(\bar{k}) = e_L(\bar{k})\) for \(\bar{k} \leq \bar{k}_L\), as implied in Lemma 7.
Now consider the case where $A_C = A_L - \epsilon$. From Lemma 7 we know that the banks’ reaction function will lie above the one for the case of independence, with $\bar{k}_C(e) \to \bar{k}_L(e)$ as $\epsilon \to 0$. Therefore, for $\epsilon$ small the equilibrium point - where $\bar{k}_C(e)$ intersects $e_C(\bar{k})$ - can be arbitrarily close to the solution under independence, but with a strictly higher level of effort.

The result that average portfolio quality improves follows: with higher equilibrium effort and an increase in the supervisory standard, $\tilde{q}_C(k) > \tilde{q}_L(k)$, along with an increase in the set of banks whose behavior is affected by the tougher standards, i.e., $\bar{k}_C - \bar{k}_C(e) > \bar{k}_L - \bar{k}_L(e)$ when $A_C < A_L$ and $\psi_C = \psi_L$, we have $\tilde{q}_C(\bar{k}_L) - \tilde{k}_C(e_C^*) > \tilde{q}_L(\bar{k}_L) - \tilde{k}_L(e_L^*)$, which implies that $\int_0^1 q_C(k) f(k) dk > \int_0^1 q_L(k) f(k) dk$ for $f(k) = 1$, as desired.
References


