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The Effects of Price Volatility and Strategic Trading Under Realization, Expected Return and Retrospective Taxation

MARK P. GERGEN*

I. INTRODUCTION

The taxation of investments in capital assets poses some of the most difficult problems under an income tax because of the deferral of returns and the volatility of prices. Recent measures address the problem of tax deferral when investments pay deferred returns by imputing income based on the expected return when the investment is made. The original issue discount (OID) rules have evolved in this direction and recent regulations on notional principal contracts and proposed regulations on contingent interest include fairly sophisticated rules that use the expected return method. Important recent scholarship has sought to extend and to refine this method. Other

* Professor, University of Texas School of Law. I would like to thank Tom Evans and Don Fullerton, as well as participants at a January 1994 meeting of the ABA Tax Section Committee on Tax Structure and Simplification and participants in a colloquium at the University of Texas for comments and questions.

1 IRC §§ 1272-1275.
2 Reg. § 1.446-3.
4 See, e.g., Noël B. Cunningham & Deborah H. Schenk, Taxation Without Realization: A "Revolutionary" Approach to Ownership, 47 Tax L. Rev. 725 (1992); Reed Shuldiner, A General Approach to the Taxation of Financial Instruments, 71 Tex. L. Rev. 243 (1992). Something akin to expected return taxation would occur if firms that undertook construction projects were required to impute income on capital devoted to the construction based on its expected return. See Thomas L. Evans, The Taxation of Multi-Period Projects: An Analysis of Competing Models, 69 Tex. L. Rev. 1109, 1143-44 (1991). One issue in the design of an expected return rule is whether income should be imputed in a way that accounts for the slope in the yield curve rather than on a constant yield to maturity method. See generally Joseph Bankman & William A. Klein, Accurate Taxation of Long-Term Debt: Taking Into Account the Term Structure of Interest, 44 Tax L. Rev. 335 (1989) (arguing that the difficulty of determining yield curves of long-term debt ex ante means that income can be measured accurately only by ex post valuations, as under a mark-to-market rule); see also Theodore S. Sims, Long-Term Debt, the Term Structure of Interest and the Case for Accrual Taxation, 47 Tax L. Rev. 313 (1992) (arguing that although the method for accruing interest can be adjusted to account for the usual upward slope in yield curves, it is not worth the effort because the effect of not accounting for sloping yield

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scholars instead have proposed taxing investments retrospectively on the basis of their actual return.\(^5\) The retrospective method taxes investments when gain or loss is realized, but allocates that gain or loss backward over the life of the investment, assuming a constant yield to maturity. Interest is charged on the deferred tax on gain allocated to prior periods and is paid on tax refunds on losses allocated to prior periods. The interest charge compensates for tax deferral.

The expected return and retrospective methods address the problem of tax deferral, but like the realization rule, they invite strategic trading—holding assets that increase in value while selling assets that decrease in value—when investment prices are volatile.\(^6\) The ability to trade strategically to improve the after-tax return has been called the "timing option."\(^7\) Eliminating both deferral and strategic trading would require a radical change in the income tax. One solution is a cash-flow consumption tax, which, by exempting investment income from tax, would eliminate any comparative advantage from deferring returns or trading assets. Another solution is to tax most capital investments on an accretion or mark-to-market basis, periodically revaluing assets to assess gain or loss.\(^8\) There has been a gradual expansion of mark-to-market accounting in recent years. Options and futures contracts that are traded on exchanges and currency futures traded in the interbank market are subject to mark-to-market accounting.\(^9\) In addition, since 1993, all noninvestment securities held by a dealer are

curves is slight as compared to other factors affecting the return on long-term debt not currently accounted for, such as changes in general interest rates).


\(^7\) E.g., Shuldiner, note 4, at 250, 255-57 (defining and explaining the timing option).


\(^9\) Regulated futures contracts, foreign currency contracts, nonequity options and dealer equity options—collectively described as § 1256 contracts—are taxed annually on a mark-to-market basis. IRC § 1256. Gain or loss is capital, with 60% long-term and 40% short-term. IRC § 1256(a)(3). Basically, this rule applies to options and futures contracts that are listed on a board or exchange and to foreign currency contracts traded in the interbank market. Equity options (options to buy or sell stock with a value determined by a stock index) are marked to market only if purchased or granted by a dealer. IRC § 1256(b). Section 1256 does not apply to an ordinary income hedging transaction. IRC § 1256(c).
subject to mark-to-market accounting. Although mark-to-market accounting is administratively difficult for assets that are not publicly traded, its application only to publicly traded assets is troubling because of the bias it creates for non-publicly traded assets. In any event, neither a consumption tax nor a broad scale accretion tax currently seems politically feasible.

The immediate prospect for taxation of investment assets is a patchwork system that uses realization, mark-to-market, expected return taxation and perhaps eventually retrospective taxation. The most likely target for innovative rules such as expected return and retrospective taxation is new financial products. This Article assesses effective tax rates and patterns of strategic trading under these four methods for investments with deferred returns and volatile prices. I use two models to evaluate each method: (1) an investment with a single payoff with binomial changes in price to establish how length of investment horizon, price volatility and trading costs affect effective tax rates and trading strategies under the four methods, and (2) a simulation of investment results under the methods using real stock data from 1960 to 1990. Both models assume no capital loss limitation

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10 IRC § 475. Equity notional principal contracts and derivatives are presumptively noninvestment securities in the hands of dealers of such securities. IRC § 475(c)(2)(D), (E). Temporary regulations clarify that sellers of nonfinancial goods and services who extend credit to purchasers are not securities dealers although they resell those notes. Temp. Reg. § 1.475(c)-1T(a). Also banks purchasing securities are not dealers if they sell only a negligible portion of those securities. Temp. Reg. § 1.475(c)-1T(b).

11 The § 1256 rule limits mark-to-market accounting to exchange-listed options and futures contracts. IRC § 1256(b), (g). This rule is problematic for it creates a significant competitive advantage for over-the-counter ("OTC") markets in multiyear options and futures contracts. This is troubling because there are economic costs to trading contracts in OTC markets, including increased credit risk and perhaps higher transaction costs (such as premiums).


13 For many investment assets, the problems that result from strategic trading may be greater than my analysis of stock data indicates. The standard deviation in total returns on common stock—a common measure of volatility—was 21.1% from 1926 to 1987 (the standard deviation in total returns on stock in small corporations over the same period was 35.9%). Ibbotson Associates, Inc., Stocks, Bonds, Bills and Inflation: 1988 Yearbook Market Results for 1926-1987, at 74 (Laurence B. Siegel & Katie B. Weigel eds., 1988) [hereinafter Ibbotson Yearbook]. These returns include dividends. Returns on commodities tend to be more volatile than returns on stock. Over the period 1961 to 1990, the standard deviations in returns on crude petroleum, silver and gold were 59.02%, 53.25% and 31.79%, respectively. Chase Global Data & Research, Chase Investment Performance Digest 34 (1991) [hereinafter Chase Digest]. With the growth of exchange trading in commodity futures, long-term plays in commodities such as gold and oil are possible at trading costs comparable to stock. The greater volatility of these investments, combined with increasing term length and lowered trading costs, are conducive to strategic trading.
and realization of gains at the terminal point of the investment horizon. The results of the two models generally are consistent.

Not surprisingly, I found that effective tax rates under the realization, expected return and retrospective methods decline as investment horizon and volatility increase and rise as trading costs increase. The most interesting finding is that effective tax rates converge under the three methods as volatility increases if trading costs are low. The stock data confirm this: For stock investments with a horizon of five years or more if trading costs are low and losses are taken monthly, effective tax rates under the expected return method approximate those under the realization rule, and effective tax rates under the retrospective method are midway between those under realization and mark-to-market. A corollary to this finding is that effective tax rates under the mark-to-market method are significantly higher than under the other methods. Rising trading costs quickly suppress strategic trading, and so increase effective tax rates under the retrospective method. Rising trading costs have more ambiguous effects under the expected return method. Strategic trading remains profitable at fairly high levels of trading costs (even in the range of 5 to 10% of asset price), but the cost of trading reduces average rates of return to close to those under mark-to-market accounting. In effect, the expected return method is neutral ex ante at high trading costs—tax plus trading costs reduce the expected rate of return by approximately the tax rate—but nonneutral ex post because strategic trading persists.

This study confirms and extends analysis by Professor Jeff Strnad of the value of the timing option for bonds taxed under the OID rules.14 Professor Strnad uses a hypothetical OID bond in an environment with fluctuating interest rates as a model of trading that is similar to

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my binomial distribution model. My study differs from Professor Strnad's in the use of real stock data, in including the retrospective method as well as the expected return method and in analyzing the effect of trading costs on strategic trading. My findings are generally consistent with those of George Constantinides' seminal study relating to the value of the timing option, though I found strategic trading to be more sensitive to trading costs than he did.

One crucial and arguably unrealistic assumption underlies this study. I assumed no capital loss limitation. The capital loss limitation tries to check strategic trading by allowing the deduction of capital losses only against capital gains (plus $3,000 per year for individuals). My assumption of no capital loss limitation may not be unrealistic, as there is good reason to believe that it is an imperfect check on strategic trading. This is suggested by studies that find a significant increase in year end trading (which is thought to be partly tax motivated) and by studies that show price volatility positively affects asset prices (which is thought to be a function of the value of the timing option). The imperfect nature of the capital loss limitation also is suggested by the availability of ways to produce risk-free capital gains to use capital losses, although such data is inconclusive. Evidence

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16 IRC § 1211.

17 See, e.g., George M. Constantinides, Optimal Stock Trading With Personal Taxes, 13 J. Fin. Econ. 65, 83-85 (1984) [hereinafter Stock Trading]; see also Dan Givoly & Arie Ovadia, Year-End Tax-Induced Sales and Stock Market Seasonality, 38 J. Fin. 171 (1983) (analyzing relationship between two phenomena in the stock market: the market return during the month of January and the existence of widespread sales of stocks toward the end of the fiscal year); Joel Slemrod, The Effect of Capital Gains Taxation on Year-End Stock Market Behavior, 35 Nat'l Tax J. 69 (1982) (revealing that sales of stocks resulting in capital losses are, ceteris paribus, significantly higher in December than in other months, and that gain taking is lower in December than otherwise would be expected). The year end effect, if there is one, is attributed to the different treatment of long- and short-term capital gains. Investors would sell losses immediately and not wait to year end if the holding period did not affect the treatment of the loss. Constantinides, Stock Trading, supra, at 83.

18 See, e.g., Andrea J. Heuson & Dennis J. Lasser, Tax-Timing Options and the Pricing of Government Bonds, 13 J. Fin. Res. 93 (1990) (finding that changes in the price of OID bonds after the 1984 adoption of the OID rules correlated positively with the volatility of bond prices). Heuson and Lasser's results suggest that the timing option is valued in the market place.

19 Section 1258 attacks a species of these—conversion transactions that produce interest-like returns in the form of capital gains. The example in the legislative history is a purchase of stock for $100 on January 1, 1994, made simultaneously with a forward con-
that strategic trading occurs or that methods exist to evade the capital loss limitation does not mean that the limitation is completely ineffective. Nevertheless, I disregard the capital loss limitation because it is difficult to model its effects. In addition, it is unclear how the loss limitation would apply under the expected return, retrospective and mark-to-market methods.\textsuperscript{20} Notwithstanding this assumption, my findings are significant. They show that a move to expected return or retrospective taxation does not eliminate the need for a rule like the capital loss limitation to check strategic trading. This in itself is an argument for the mark-to-market method over the expected return or retrospective methods because the capital loss limitation has troubling distributive consequences and drives much tax planning.

Other assumptions I made are less crucial or unrealistic. I assumed investments would be sold and gain recognized at the terminal point on the investment horizon. Individual investors often hold investments that appreciate significantly in value until death, thereby avoiding any tax on gain because of the basis step up.\textsuperscript{21} This possibility of complete tax avoidance on gain reduces the effective tax rate on investments by individual investors. Presumably, this effect is more significant under the retrospective method than the expected return method.\textsuperscript{22} The assumption of a fixed investment horizon holds for astract to sell the stock for $115 on January 1, 1996. Senate Comm. on Finance, 103d Cong., 1st Sess., Fiscal Year 1994 Budget Reconciliation Recommendations of the Committee on Finance at 102 (Comm. Print 1993). Transactions or positions that produce interest-like returns are difficult to identify. The statute lists the two most obvious cases—(1) buying an asset while simultaneously entering into a forward contract to sell and (2) straddles—and tries to cover others by including “any other transaction which is marketed or sold as producing capital gains from” an interest-like return, IRC § 1258(c)(2)(C), and empowering the Treasury to include other transactions by regulation, IRC § 1258(c)(2)(D).

\textsuperscript{20} Character, and thus the applicability of the loss limitations, differs under the two mark-to-market rules. Gain or loss on noninvestment securities in a dealer’s hands is ordinary, IRC § 475(d)(3)(A)(i), while gain or loss on exchange-listed options and future contracts is capital, IRC § 1256(a)(3). Applying the capital loss limitation under retrospective taxation raises many questions. For example, is a loss that is carried back usable only if there are sufficient gains in the carryback year? If there are unused losses and later gains that carry back to the year to which the loss is carried back, may the loss shelter the gains? The proposed regulations on contingent interest treat both contingent payments in excess of the projected amount and gain on the sale or retirement of an instrument as ordinary income. Prop. Reg. § 1.1275-4(b)(6)(ii), -4(b)(8)(i), 59 Fed. Reg. 64,884, 64,896, 64,898 (1994). Contingent payments that are less than the projected amount and loss on the sale or retirement of an instrument give rise to an ordinary loss to the extent of prior interest inclusions. Prop. Reg. § 1.1275-4(b)(6)(iii)(B), -4(b)(8)(ii), 59 Fed. Reg. 64,884, 64,896, 64,898 (1994). Excess negative adjustments during the life of an instrument may be carried forward to offset against future interest income. Prop. Reg. § 1.1275-4(b)(6)(ii)(C), 59 Fed. Reg. 64,884, 64,896 (1994). Only losses in excess of prior interest accruals are capital and subject to the loss limitation.

\textsuperscript{21} See IRC § 1014(a).

\textsuperscript{22} Gains at normal return rates in excess of dividends paid would not escape tax under the latter method.
sets like bonds or options with fixed payout or expiration dates, but it may not hold for assets like stock or real estate with perpetual lives.

Furthermore, I assumed a single tax rate with no capital gains preference. Presumably the capital gains preference reduces the effective tax rate on investment income. I also assumed no wash sale rule. The wash sale rule probably is not a significant check on strategic trading since it easily is avoided by delaying for 31 days the repurchase of a security or by purchasing a nominally different security with similar characteristics.

Care must be taken in deriving tax policy conclusions from this study. Even if the models accurately depict real world investment strategies, the normative implications of the findings are not transparent. Usually tax policy is driven by concerns of neutrality, equity and administrability. One might conclude from my findings that a mark-to-market rule is preferable to the other rules on grounds of neutrality and equity. Mark-to-market accounting is more neutral ex ante because the expected effective tax rate on investments is closer to

23 The maximum capital gains rate for individuals is 28%, IRC § 1(h), while the maximum individual rate is around 40% (39.6%, IRC § 1, coupled with the effects of the phaseout of the personal exemption, IRC § 151, and the limitation on itemized deductions, IRC § 68).

24 See IRC § 1091 (disallowing loss from wash sales of stock or securities).

25 An income tax is said to have a neutral effect on investment if it reduces the pretax expected return by an investor's tax rate when an investment is made and throughout its life regardless of the timing of returns or risk. This is Professor Samuelson's standard for neutral depreciation expanded to cover the entire life of an investment. See Paul A. Samuelson, Tax Deductibility of Economic Depreciation to Insure Invariant Valuations, 72 J. Pol. Econ. 604 (1964). Under a neutral tax, those with different tax rates would value investments the same, so there would be no tax-motivated trading of assets.

Neutrality does not mean that tax will not affect investment. Even a neutral income tax diminishes incentives to invest by reducing the return. An income tax promotes risky investments by those who are risk averse because it reduces the variance on returns by taxing gains while giving a tax refund on losses. Nor is a neutral tax necessarily allocatively efficient. Some non-neutral rules impose transaction costs, but do not alter resource allocation. This is true, for instance, of the sale and repurchase of an investment to realize a loss. Furthermore, to the extent the market cannot capitalize the value of a tax preference into the price of an asset (increasing the price so that the after-tax expected rate of return is the same as that of nonpreferenced assets), a preference will have a distributive effect (investors will get higher than normal rates of return), but not necessarily an allocative effect. This is the familiar point that capitalization of preferences drives out inequity since buyers of preferred assets get no better than normal returns while inviting inefficiency since preferences are capitalized—that is, prices drop—because of the increased supply of the preferred asset to meet the tax-driven demand. See Boris I. Bittker, Equity, Efficiency, and Income Tax Theory: Do Misallocations Drive Out Inequities?, in The Economics of Taxation 19 (Henry J. Aaron & Michael J. Boskin eds., 1980). Even an increase in supply of a preferred asset that results in capitalization of the preference may not be inefficient. A preference may offset some other factor distorting the allocation of resources. For instance, current tax law prefers long-term investments with volatile returns, which may not be undesirable if investors are more risk averse and shortsighted than is socially optimal.

26 "Equity" loosely means taxing people according to their relative ability to pay tax.
the nominal tax rate; it is more neutral ex post because it suppresses strategic trading. Mark-to-market accounting arguably is more equitable because investors with either winning or losing investments bear tax in closer proportion to their gains or losses. The analysis, however, is not that simple. Neutrality is a proxy for efficiency; it is debatable whether increasing the effective tax rate on investment is socially desirable, or whether moving the effective tax rate closer to the stated tax rate for some investments will have a beneficial effect when that increase in the effective tax rate is mediated through other imperfections in the tax system or the financial world. Indeed, taxing a subset of investments under a mark-to-market rule could have a harmful effect, for the significant difference in effective tax rates under the mark-to-market and other methods creates a significant tax discontinuity that is likely to roil activities in financial markets near the boundary of mark-to-market accounting.\(^2\) The equity argument is even more dubious. Equity considerations usually do not figure significantly in the technical assessment of tax rules. Markets may adjust to rules so that differently taxed investments have similar after-tax returns, making it difficult to claim that investors are taxed inequitably. Even if not capitalized, the effect of technical differences in rules on returns are often so slight that they do not easily connect to the poorly defined norms of distributive justice that underlie the concern with equity. It is very difficult to make distributional arguments for adoption of mark-to-market accounting because it is unknown who would bear the cost of the increase in the effective tax rate on investment from such a move. The problem is akin to that of predicting who bears the cost of the double tax on corporate income, which has proven insoluble to this point.\(^2\) Moreover, changes should be avoided, since they create windfall gains and losses that are capricious.

Nevertheless, this study does have important policy implications. First, it suggests that the expected return or retrospective methods should not apply to assets like publicly traded stock with volatile prices and low trading costs because once the possibility of strategic trading is taken into account these methods differ little from the rule of realization in the effective tax rate they impose on investment. This

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\(^2\) Consider the tale of Market Index Target-Term Securities (MITTs) and Stock Market Annual Reset Term (SMARTs) Notes. These are exchange-listed notes that pay principal plus interest contingent on performance of the Standard & Poor index. MITTs fared poorly in the market because taxable investors feared they would be subject to mark-to-market accounting under § 1256. This fear prompted Merrill Lynch to develop SMART Notes because the reset feature made it largely irrelevant whether the contract was marked to market. See Freeman & Lipton, note 13, at 178-81. The expense of designing § 1256-resistant instruments like SMART Notes is a deadweight loss.

point is important because regulations on notional principal contracts\textsuperscript{29} and the proposed regulations on contingent interest\textsuperscript{30} use the expected return method. Whatever equity and economic gains that may result from moving the effective tax rate on investments marginally closer to the stated tax rate through expected return or retrospective taxation probably are outweighed by the greater administrative costs. These include the cost of collecting the additional data needed to assess tax under the methods (the most important new data are the imputation rate under the expected return method and the lending rate under the retrospective method), the cost of making additional calculations of greater complexity to assess tax, and the costs tax advisors incur in learning the methods and devising strategies to best them.

The study also suggests that it may be ill-advised to apply the expected return method to investments with significant trading costs because the method stimulates strategic trading, with a significant increase in tax-induced trading costs. This conclusion is more tentative than the first, because the profit from strategic trading shrinks as trading costs rise, and so the capital loss limitation with even a muted effect on psychological factors such as an unwillingness to incur hard costs to reap marginally greater tax savings may suffice to deter strategic trading.

These suggestions are negative. On a more positive note, the Article suggests a possible three-pronged strategy for investment taxation. Assets that trade at very low costs would be subject to mark to market taxation. This would include most publicly traded assets and certain over-the-counter financial contracts with low trading costs. The retrospective and expected return methods offer little improvement over the realization method in the taxation of such assets because of the problem of strategic trading. Assets that trade at moderate but non-prohibitive costs would be taxed under the retrospective method. Moderate levels of trading costs seem to suppress strategic trading under the retrospective method. Further, the effective tax rate that an investment can be expected to bear under that method approximates that under mark-to-market taxation (unless the asset price is extremely volatile, in which case the effective tax rate may be less even than that under the realization method). Liquidity and valuation concerns make mark-to-market taxation unattractive for non-publicly traded assets, and the expected return method is a poor alternative because it may induce costly strategic trading. Assets that trade at prohibitive costs would be taxed under the expected return method.

\textsuperscript{29} Reg. § 1.446-3.
If strategic trading is impractical, the effective tax rate that an investment can be expected to bear under the expected return method approximates that under mark-to-market taxation. Liquidity and valuation concerns make the mark-to-market method impractical for such assets, and the expected return method may be preferable to the retrospective method because it is less complex, it results in an effective tax rate nearer to that under the mark-to-market method than the retrospective method (particularly if prices are very volatile) and it lessens the credit risk to Treasury. This three-pronged strategy makes it possible to abolish the capital loss limitation because it addresses the problem of strategic trading directly by adopting tax rules that minimize the likelihood of strategic trading. Although this strategy involves three different regimes for taxation of investment assets, with the attendant problems of boundary drawing, it provides a coherent theoretical basis—asset liquidity—for drawing boundaries.

II. Two Responses to the Problem of Deferral: Expected Return and Retrospective Taxation

Deferral of tax on income is equivalent to an interest-free loan from the government of the tax due on the income. Thus, long-term investments, such as stock and collectibles, which may provide deferred returns through appreciation, are tax-advantaged. The benefits of tax deferral are well-known. This Section describes two responses.

A. Expected Return Taxation

Much theory and recent law on the taxation of investments with deferred returns attempts to solve the problem of deferral by imputing income over the life of the investment on the basis of its expected return when the investment is made. The OID rules have evolved toward the expected return method in an effort to account properly for implicit interest in zero interest bonds. If an investor pays $700

\[ \text{Deferral} = \frac{t}{(1 + r(1 - t))^n} \]

Thus, a 40% tax declines to a 37.6% tax if deferred one period at a 10% interest rate. The after-tax rate of return increases from \( r(1 - t) \) to \( r(1 - t(1 + r(1 - t))^n) \), or from 6% to 6.24% under these assumptions.

\[ \text{See notes 2-4. The generally accepted theoretical model for depreciation—economic} \]
\[ \text{depreciation—also seeks to tax expected return. See William D. Andrews, Basic Federal} \]
\[ \text{Income Taxation 810 (4th ed. 1991) (describing economic depreciation). This model} \]
\[ \text{depreciates an investment over its life by assuming that the expected return and the discount} \]
\[ \text{rate do not change over the life of the asset. Thus, an investment of $1,000 that is expected} \]
\[ \text{to earn $400 per year for three years, at a 9.7% discount rate, will decline in value to $697} \]
\[ \text{in the first year because that would be the value of the investment after one year if two} \]
\[ \text{$400 payments remained and the discount rate remained 9.7%}. \]

\[ \text{See IRC §§ 1272-1275.} \]
for a bond that pays $1,000 in five years, $300 is treated as interest and accrues over the life of the bond at a rate of 7.4% per year compounded annually. This is not true expected return taxation because 7.4% per year would represent the expected return on the bond only if it were not subject to credit risk or interest rate risk. An OID bond subject to a constant credit risk will increase in value at a slower rate than the imputed interest rate. Interest rate risk, which is reflected in the yield curve, means that the expected rate of increase in the value of the bond differs from simple compound interest.

True expected return taxation appears in the attempt to account for the interest-like element in bonds or other instruments with contingent payments. If an investor pays today for the right to a contingent payment at some future settlement date, the contingent payment has an interest-like element that is reflected in the difference between what the investor paid and what he would have promised to pay on the settlement date for that same contingent payment in the cash forward or futures markets. A cash forward price is the price a buyer agrees to pay and a seller agrees to accept on the settlement date for a specified commodity (or, if it is an interest rate forward, the agreed interest rate on a loan or deposit to be made in the future). Forward contracts are settled in cash for the difference between the forward price and the spot price of the commodity. Generally, cash forward prices rise in an interest-like fashion over time; otherwise, an opportunity for arbitrage would exist. For example, if cash forward prices for a commodity rise at a rate faster than current interest rates, riskless profit could be made by borrowing to purchase the commodity and making a forward contract to sell it. The profit is the rate difference less the commodity's carrying cost.

34 For a particularly dramatic example of this, consider a purchase for $300 in the secondary market of a deeply discounted bond, which pays an above-market rate of interest and $1,000 in 10 years. If the bond is modified in a significant respect, accrual of that $700 over the 10-year life of the bond is required. See John E. Capps, Note, In the Wake of Cottage Savings: The Tax Consequences of Debt Modifications, 72 Tex. L. Rev. 2015, 2022-23 (1994) (discussing tax consequences of OID rules). Such accruals significantly overstate the income on the bond over its life. They impute income assuming an increase in value of the bond to $1,000 at maturity. In fact, the bond's value may decrease over time because of unresolved insecurity about payment of principal and the dwindling stream of premium interest payments.

35 See Bankman & Klein, note 4, at 338. In the very recent past, the yield curve had a steep upward slope. In this environment, a long-term fixed interest bond could be disaggregated into a series of cash forward contracts for loans with steeply rising interest rates, resulting in interest accruals at a rate slower than simple compound interest.

36 See Shuldiner, note 4, at 269-72, 287-89.
Proposed regulations on debt instruments with contingent payments employ the expected return method. They require an issuer and a holder to construct a schedule of payments by predicting the value of contingent payments as of the instrument's issue date. The projected yield of the instrument is calculated based on this schedule, and interest is accrued on a compound basis by multiplying the instrument's adjusted issue price by its projected yield. If possible, contingent payments are valued using forward prices for the underlying property.

**Example 1:** An instrument issued on December 1, 1994, at a price of $1,000 entitles the holder on December 1, 1999 to $1,000 multiplied by the percentage change in the price of gold from December 1, 1994 to December 1, 1999. The spot price of gold on December 1, 1994, is $400 an ounce. On the same date, the forward price of gold with a December 1999 contract is $561 an ounce. The expected payment on the

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37 Prop. Reg. § 1.1275-4(b)(5), 59 Fed. Reg. 64,884, 64,896 (1994). These regulations are the fourth set of regulations dealing with contingent interest in the last eight years. In January, 1993, regulations on contingent interest were proposed, but withdrawn pending review by the incoming Clinton Administration. Prop. Reg. § 1.1275-4 (filed with the Federal Register Jan. 19, 1993 in proposed form, but withdrawn before official publication by the Office of Management and Budget). The 1993 regulations also employed the expected return method, but, in addition, gave issuers and holders an option to elect to be taxed under a method akin to the mark-to-market method (the spot price method) and under a method that was an odd hybrid of the mark-to-market and expected return methods (the yield adjustment method). Good explanations of the 1993 regulations are found in David C. Garlock, A Primer on the New Proposed (Almost) Regulations for Contingent Debt Instruments, 58 Tax Notes 1225 (Mar. 1, 1993); David P. Hariton, Contingent Debt: Putting the Pieces Together, 58 Tax Notes 1231 (Mar. 1, 1993). The 1994 regulations eliminate these options. An earlier set of proposed regulations, published in 1991, bifurcated investments with contingent payments into contingent and noncontingent components and required imputation of interest on the noncontingent component. Prop. Reg. § 1.1275-4(g), 56 Fed. Reg. 8310 (1991). An even earlier set of proposed regulations, issued in 1986, imputed interest on a bond with a contingent payment only to the extent noncontingent payments on the bond exceeded the issue price. Prop. Reg. § 1.1275-4(e), (f), 51 Fed. Reg. 12,022 (1986). In effect, no effort was made to accrue interest on the contingent component prior to realization. The New York State Bar Association, while critical of many specific points in the regulations, accepted expected return taxation in principle for instruments with substantial deferred payments. N.Y. St. Bar Ass'n Tax Sec., Report and Recommendation for the Treatment of Contingent Debt Instruments Under Proposed Regulations 1.1275-4, reprinted in 61 Tax Notes 1241 (Dec. 6, 1993) [hereinafter NYSBA Comments].


contract is $1,402. The yield is 7%. The issuer and holder will accrue interest at a 7% compound rate on the instrument.\footnote{Under the 1991 regulations, the instrument would have been bifurcated into a purchase of a $1,000 five-year OID bond, with an issue price equal to the discounted present value of such a right at the current applicable federal rate ("AFR") ($713 at a 7% discount rate), and a forward contract on gold. Interest would have been imputed only on the bond component. Thus, $287 of interest would have been imputed, rather than $403. Under the 1986 regulations, no interest would be imputed on the instrument because the issue price equals the noncontingent payment.}

If forward prices are not readily available, contingent payments are estimated by determining a reasonable rate of return on the instrument based on the returns on comparable instruments and then constructing a payment schedule consistent with that yield.\footnote{Prop. Reg. § 1.1275-4(b)(4)(ii), 59 Fed. Reg. 64,884, 64,894 (1994). The AFR is a floor.}

The expected return method also is used in the notional principal contract regulations.\footnote{Reg. § 1.446-3.} Notional principal contracts are instruments such as swaps where parties periodically exchange payments and one or both payments is calculated by applying an index to a notional principal amount. The regulations attempt to isolate contracts with significant deferred returns in the definition of nonperiodic payments,\footnote{"Nonperiodic payments" are payments other than "periodic payments" as defined in Reg. § 1.446-3(e)(1).} and require that income and expense with respect to such payments be calculated on an expected return basis. An example is an up front payment for an "in-the-money" swap (that is, a swap that has positive net present value to a party because the expected return exceeds the expected cost).\footnote{Reg. § 1.446-3(f)(4) (Ex. 7).}

The regulations recast the payment as a loan to the up front payee with the expected payouts (interest and principal) to the up front payor computed using cash forward prices. Interest is imputed in these cash flows assuming a constant yield to maturity. This interest is backed out to derive the principal allocable to each period, which the up front payor treats as a payment made and the up front payee treats as a payment received in that period.\footnote{The interest is disregarded (in theory, the up front payor should have imputed interest income and the up front payee should impute interest expense) because interest paid and received would wash out each period as counter payments on the swap.}

Under the expected return method, contingencies that affect actual return are not accounted for as they occur. Gains or losses from such contingencies are recognized when an investment is disposed of, or are accrued implicitly over time as the investor earns supranormal or subnormal returns by holding an investment after a change in its
value. Therefore, in Example 1, interest continues to accrue at a 7% rate even if the December, 1999 forward price drops. Gain or loss is recognized in Year 5 or when the contract is sold. Similarly, interest accrues on an OID bond regardless of changes in interest rates or credit worthiness of the issuer. Such changes affecting the value of the bond produce taxable gain and loss when the bond is disposed of, or if the bond is held, gradually accrue over time as the holder imputes interest at above or below the market rate. 48

B. Retrospective Taxation

Another response to the problem of deferral is to determine the gain or loss on an investment when it pays off, allocate that gain or loss over the term of the investment at a compound rate assuming a constant yield to maturity and charge interest on the tax on the gain attributed to earlier periods at some interest rate.

Example 2: B invests $1,000 for a right to $1,000 times one plus the percentage increase in the Dow over five years. The tax rate is 40%. The Dow doubles in five years, so the investment's end value is $2,000 and the annual rate of return is 14.87%. Under retrospective taxation, income accrues at a rate of 14.87% compounded over the five years. 49 Tax is paid at maturity, but with interest charged on the tax attributed to Years 1 through 4. If the interest rate were 6%, B would have $59.48 tax attributed to the first year 50 and owe an additional $15.63 interest on that tax. 51

The rules on Passive Foreign Investment Companies ("PFICs") employ a simplified form of retrospective taxation to combat tax deferral on income accumulated in foreign investment funds. 52 Gains on the

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48 Assume A invests $700 in a bond paying $1,000 in five years, a 7.34% discount rate. A accrues $51 interest in the first year. After one year, the discount rate drops to 6%. The value of the bond rises to $792. If A sells the bond, he would immediately recognize $41 gain ($792 − ($700 + $51)). If he retains the bond, he accrues $55 interest, although if the discount rate remains 6%, the value of the bond would increase only $48 to $840.

49 If I is the investment, v is the end value and n is the number of periods the investment is held, the rate of return = (v/i)^n − 1.

50 The income is $1,000 × 14.87% or $148.70 and the tax is 40% × $148.70 or $59.48.

51 Interest on $59.48 at 6% for four years is $15.63.

52 Some features of retrospective taxation appear in other parts of current law. In some installment sales, compound interest is added to deferred installment gain when the gain is recognized, though this rule differs from the retrospective method in that the gain is treated as realized on sale. IRC § 453(b)(3). Dealers who sell residential lots or timeshares may elect this method. Other provisions charge interest on deferred tax liabilities, though they differ from retrospective taxation in that the gain and the deferred tax are calculated and the interest is paid currently. IRC § 453A(c) (charging interest on certain deferred
disposition of stock in a PFIC or on excess distributions from a PFIC are prorated ratably over the holding period of the PFIC stock as ordinary income, tax is assessed on the prorated income at the highest individual or corporate rate for each year to which income is attributed, and interest is levied on deemed tax underpayments in prior years. This approach is simplified because it apportions gain on a ratable (that is, a straight-line) basis. Further, it does not permit retrospective allocation of losses with interest paid by Treasury on deemed tax overpayments in prior years. The method of retrospective taxation analyzed in this Article determines the yield on an investment on disposition and prorates income assuming a constant yield to maturity. It also treats losses consistently with gains allocating losses retrospectively and paying interest on deemed tax overpayments.

Retrospective taxation differs from expected return taxation in several obvious respects. Under the retrospective method, tax on the expected return is collected at the end rather than over the life of an investment. The difference in timing is made up by charging interest on taxes attributed to prior periods, but there is a greater credit risk to the government under the retrospective method. The method also differs from taxing expected returns because it effectively taxes supernormal or subnormal returns over the life of the investment rather than at the end. In Example 2, expected return taxation would tax any excess over the expected yield on the investment in Year 5. Retrospective taxation prorates and taxes such gain over the term of the investment.

Alan Auerbach has suggested a different form of retrospective taxation. This approach taxes investments on disposition based on their

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installment gain to nondealers), § 995(f) (charging interest on unrepatriated DISC income). In addition, a look back rule requires that firms that over- or underestimate income on a multiyear construction project make retrospective adjustments to income included under the percentage of completion method and pay or receive interest on resulting tax under- or overpayments. IRC § 460(b)(2).

An “excess distribution” is a distribution that exceeds 125% of the average actual distributions received in the three prior years. IRC § 1291(b).

IRC § 1291(a)(1)(A), (B).

IRC § 1291(a)(1)(C), (c).

Interest is assessed from the due date of the return for the year to which the income is attributed at the normal rate for underpayments (the federal short-term rate plus 3%). IRC § 1291(c)(3).

Ratable apportionment tends to front load income because it ignores compounding. Ratable apportionment simplifies more than just the mechanical steps of apportioning income backwards. Under the form of retrospective taxation analyzed in this Article, all returns on an investment must be recorded in order to calculate the yield to date. Yield need not be calculated for ratable apportionment. Furthermore, when income is apportioned retrospectively based on the calculated yield, I subtract previously taxed returns to determine the as yet untaxed yield attributable to each year. No such adjustments need be made under the PFIC rules.
end value and an assumption that this amount includes income accrued over the period the investment was held at the normal rate of return for that period. 58

Example 3: An investment on January 1, 1992 returns $1,000 on January 1, 1993. The normal annualized rate of return for the year was 10%. The investment is assumed to have earned income at that rate over the year to reach an end value of $1,000, or $90.91. This amount is imputed and taxed as income whatever the original amount of the investment.

This method is perfectly neutral with regard to the decision to hold or to sell an investment with accrued gains or losses, assuming the expected return on an investment for the next period always equals the normal rate of return. 59 It is easier to administer than the expected return method and the retrospective method, because it requires one less item of information (the amount invested need not be known). The method may be objectionable on equity grounds, however, because tax is independent of actual income. Thus, the tax would be the same in Example 3 whether the $1,000 return was mostly winnings on an investment that paid off handsomely or the remaining value of an investment gone bad.

C. Imputation and Lending Rate

One of the more difficult administrative problems under the expected return and retrospective methods is setting the rates for imputing income or charging interest on deferred taxes. 60 The rates under

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58 See Auerbach, note 6, at 168. The actual formula for assessing the tax for an investment with a single return of \( A \) where \( t \) is the tax rate, \( i \) is the risk-free interest rate and \( s \) is the period the investment is held is \( (1 - e^{-st})A \). \( e \) is the natural logarithm, approximately 2.718.

59 Id. at 168.

60 Two features of the contingent interest and the notional principal contract regulations attempt to reduce administrative costs. First, they preserve the rule of realization for contracts with stated returns, paid or accrued, that should approximate the expected return. The contingent interest regulations do this through an exception for a variable rate debt instrument (“VRDI”). Prop. Reg. § 1.1275-4(a)(2)(ii), 59 Fed. Reg. 64,884, 64,893 (1994). VRDIs pay or accrue interest currently, either at a floating rate that is expected to track the cost of newly borrowed funds or at a single objectively determined rate. Reg. § 1.1275-5(a). Subject to a de minimis rule, VRDIs also must provide for noncontingent principal repayments equal to the issue price of the contract. Reg. § 1.1275-5(a)(2). The contingent interest regulations extend the variable yield method to cover debt instruments that do not satisfy this rule if the instrument provides for a single contingent payment at maturity with a readily determinable forward price. Prop. Reg. § 1.1275-4(b)(4)(iii)(A), 59 Fed. Reg. 64,884, 64,894 (1994).

The notional principal contract regulations preserve the rule of realization for periodic payments. Periodic payments, which are payments made at least annually over the life of
the two methods may differ. The interest charge on deferred taxes (“the lending rate”) should be based on the taxpayer's cost of borrowing; the rate used to impute income under the expected return method should be based on returns available to the taxpayer. The lending rate often should exceed the risk-free market rate of interest (for example, the rate on U.S. government securities) because of greater default risk by private borrowers. The imputation rate more often will approximate the risk-free market rate, but the imputation rate may be higher since investments may provide higher returns because of nondiversifiable risk and because some investors may be able to command higher than normal returns.

Neither the imputation rate nor the lending rate can be determined with perfect accuracy. Generally applicable rates may be set administratively, as they are under current law through the AFR. Such generally applicable rates ignore differences in investors and investments. All taxpayers are charged the same rate on deferred taxes, regardless of variations in their credit risk. Income is imputed on all investments at the same rate regardless of differences in expected rates of returns due to nondiversifiable risk or other factors. Thus, if deferred payments on a sale of property are subject to credit risk, imputing interest based on the AFR understates interest and overstates principal on the note.

The notional principal contract regulations also offer a simplified method for computing expected return. Nondealers may use the alternative of the “level payment method” for accounting for nonperiodic payments. Reg. § 1.446-3(f)(2)(iii)(A). For an up-front payment for an in-the-money swap, one derives the stream of level payments over the life of the swap with the present value of the up-front payment using a reasonable rate. The principal part of each of these payments is treated as a payment made and received on the swap in the period to which it relates. Id.

The New York State Bar Association suggested further simplification by extending VRDI treatment to contracts that pay or accrue interest currently at a single formula over the life of an instrument and have no other contingent payments, even if the issue price exceeds noncontingent principal. Furthermore, they would simplify the measure of expected return by imputing return at the AFR on the adjusted basis of contracts. NYSBA Comments, note 37, at 1245.

61 The AFR is set monthly by the Service based on yields on U.S. government securities. IRC § 1274(d)(1)(C). The AFR is used for several purposes, including imputing interest on low interest notes given for property, IRC § 1274, and on low interest loans, IRC § 7872, and (indirectly) determining the interest charge on taxes deferred on gain on large installment notes. IRC § 453A(c)(2)(B) calculates interest based on the tax underpayment rate, which is 3% above the AFR. IRC § 6621(a)(2).

62 If the asset sold is nondepreciable and the installment sale rules apply, such misstatement harms the buyer (who loses an interest deduction) and benefits the seller (who defers income until payment on the note). The misstatement, in the aggregate, may cost the parties more tax than it saves because the buyer’s deduction is deferred longer than the
Rates also can be set on a more individualized basis. The proposed contingent interest regulations require that the projected value of future contingent payments be set using forward prices of the underlying property.\(^6^3\) If such information is not available, the expected return is based on the returns offered on comparable instruments taking into account such factors as the market rate of interest and the credit quality of the issuer.\(^6^4\) The AFR is used only as a floor on the projected return. The lending rate similarly could be derived from a taxpayer’s cost of borrowing or from the cost of borrowing of comparable firms. Such approaches permit closer tailoring of rates, but are more costly to administer and invite self-serving assessments of rates.

**D. Investments With Multiple Payoffs**

The expected return and retrospective methods can be adapted to instruments with multiple payoffs, though sometimes with difficulty. One possibility is to disassemble an instrument with multiple payoffs into a series of instruments with single payoffs to which each method would apply as above.

*Example 4:* \(C\) invests $1,000 in an instrument that repays $1,000 in five years plus interest at the end of each year equal to 70% of the increase, if any, in the S&P 500 Index in that year. This instrument would be divided into five components, one for each year’s payment (the last year’s component would include principal and a contingent payment). The $1,000 issue price would be allocated among the components based on their value on the issue date. Under the expected return method, income would be imputed separately on each component, and additional gain or loss would be recognized separately on each component when it matured. Under the retrospective method, tax would be assessed based on the yield of each component as it matured.

If done properly, this approach can improve accuracy. In the example, disaggregation would permit the investor to realize immediately the loss if any payment was less than expected. Without disaggregation, any loss would be deferred until the instrument matured or was sold. Disaggregation also makes it possible to impute a higher yield on more remote payments consistent with the usually rising yield

curve. This approach, however, is susceptible to abuse in allocating the purchase price among components.\textsuperscript{65}

There are simpler approaches to taxing dividend or interest-like payments that do not diminish the year-to-year value of the underlying instrument. Under the expected return method, dividend or interest payments can be taxed and income imputed to the extent the expected return exceeds the payment.

\textit{Example 5:} In \textit{Example 4}, if the expected rate of return when the instrument was acquired were 10\% and $12 were paid in Year 1, the $12 would be treated as interest and would be taxed. An additional $88 of income would be imputed. In Year 2, if $14 were paid, that $14 of interest would be taxed and $94.80 of income would be imputed.\textsuperscript{66}

Payments in excess of the expected return might be fully taxed with no effect on imputation in future years; they might be taxed up to the amount of the expected return with any excess treated as a recovery of basis; or they might be fully taxed with any excess over the expected return credited against future imputation. I adopt the first approach, which is the harshest to the investor.\textsuperscript{67}

Under the retrospective method, if dividend or interest payments are reinvested in the same account, the simplest approach is to ignore the payment and tax the eventual cash payout from the account retrospectively. This approach increases the government’s credit risk and creates a short-term revenue loss. In addition, this approach does not work for payments that are not reinvested in the original account.\textsuperscript{68}

An alternative is to tax the payment, calculate the investment’s internal rate of return using the payment and the end return from the investment, and impute income retrospectively using that internal rate.

\textsuperscript{65} The proposed regulations on contingent payment debt instruments adopt an approach akin to that in \textit{Example 4}. A payment schedule that forecasts each contingent payment separately is constructed and adjustments are made as each contingent payment is determined. Prop. Reg. § 1.1275-4(b)(2). Positive adjustments are treated as interest. Prop. Reg. § 1.1275-4(b)(6)(ii). Negative adjustments are deductible, though only to the extent of past or current interest inclusions. Prop. Reg. § 1.1275-4(b)(6)(ii). This is similar to the approach described in \textit{Example 4} except that a loss may be taken only against interest inclusions. Thus, in \textit{Example 5}, the consequence of receiving a payment of $12 in Year 1 assuming the forecasted payment was $100 is that the holder would include $12 in income. The holder could not realize a loss on the part of the issue price associated with the Year 1 payment.

\textsuperscript{66} The expected return would be $108.80 ($1,088 x 10\%).

\textsuperscript{67} See Appendix.

\textsuperscript{68} This approach also assumes no capital gains preference.
of return, but giving a credit for taxes already paid. I adopt this second approach.\textsuperscript{69}

Rules like these that fully tax dividend or interest-like payments require some mechanism to determine when a payment reduces the year-to-year value of an instrument, or in tax parlance, represents a recovery of basis. In my study of stock data, I followed the conventions of the corporate income tax in determining what was a dividend and what was a recovery of capital.\textsuperscript{70}

III. The Impact of Volatile Prices: The Persistence of Strategic Trading Under the Expected Return and Retrospective Methods

Neither expected return taxation nor retrospective taxation neutrally taxes investments with volatile prices. Indeed, for highly volatile liquid investments, like stock, effective tax rates under these methods are comparable to those under the realization rule.\textsuperscript{71} This is due to strategic trading that is made possible by the mistaxation of supranormal and subnormal returns. Expected return taxation defers recognition of non-normal returns; retrospective taxation blends returns that may fluctuate over the life of an investment to impute returns at a single rate.

The findings and analysis in this Section are based on two models that simulate the effect of alternative tax rules on investments with volatile prices. One model employs a hypothetical investment with binomially distributed returns each period with an average return equal to an assumed normal rate of return.\textsuperscript{72} The other model uses monthly price and all distribution data for 500 stocks for the period from 1960 to 1990 drawn from the Center for the Research in Security Prices ("CRSP") data base.\textsuperscript{73} It tracks the outcome of an investment

\textsuperscript{69} See Appendix.

\textsuperscript{70} See IRC §§ 301(a), 316. Minor adjustments had to be made in these rules under the retrospective method. See Appendix.

\textsuperscript{71} My use of the effective tax rate to compare the effect of alternative tax rules on an investment is problematic. The calculation of the effective tax rate is explained in the Appendix and its use as a basis for comparison is justified in note 77. Unless otherwise stated, by effective tax rate, I mean the tax rate that an investment is expected to bear when it is made. This is determined by comparing the rate of return on the average or expected return on an investment pre- and post-tax.

\textsuperscript{72} In this model, an investment may rise in value by a factor of \(x\) with a probability of \(PR(x)\) or decline in value by a factor of \(y\) with a probability of \(PR(y)\) each period where \(xPR(x) + yPR(y) = r\), or the normal rate of return expressed as a factor.

\textsuperscript{73} The files include stocks from the New York Stock Exchange, the American Exchange and NASDAQ. The CRSP data files were developed at the Graduate School of Business at the University of Chicago. For a description and history of the files, see Center for Research in Security Prices, CRSP Stock File Guide (1991). I obtained the data through the University of Texas Computation Center.
of $10,000 in a stock under each of the methods. The two models have different strengths and weaknesses, but they produce generally consistent results. The models are explained in detail in the Appendix.

A. Absolutely Illiquid Investments

I start with an absolutely illiquid investment—an investment that must be held until the end of the life expected at its purchase—because it aptly illustrates how the expected return and retrospective methods can mistax investments with returns higher or lower than the average rate of return. The expected return method does this by deferring recognition of gains and losses from such non-normal returns. The retrospective method does this by blending returns that vary over time. If high returns precede low returns, blending returns effectively defers tax on part of the high returns. Conversely, blending returns accelerates tax when low returns precede high returns. Under the expected return method, the effects of undertaxing an investment if the return is supranormal while overtaxing it if the return is subnormal will offset each other so that the effective tax rate that an investment can be expected to bear on the average return will equal the stated tax rate (assuming the imputation rate equals the expected rate of return). These effects do not cancel out under the retrospective method. The consequence is that an increase in the volatility of an investment decreases the effective tax rate under the retrospective method even without strategic trading.

1. Binomial Distribution Mode

In this Article, I am concerned mostly with the effect of different tax rules on the expected after-tax return on an investment when it is made. But it is useful to start by comparing the tax burden under the rules on different outcomes. A few basic points can be made with the binomial distribution model using a two-period investment. This investment can take four possible paths to three possible pretax end values in period two. I denominate a high return as \( x \) and a low return as \( y \). Without strategic trading, under the realization, expected return and retrospective methods, the tax burden is solely a function of the end value and is not affected by the path. The tax burden is a function of both the path and the end value under the mark-to-market method. The realization and expected return methods have the lowest ex post effective tax rate on the path with constant high returns (the \( xx \) path).
because the most gain is deferred for the longest time on that path.\textsuperscript{74} Conversely, they bear the highest ex post effective tax rate on the path with constant low returns (the \textit{yy} path). Under the retrospective method, the ex post effective tax rate will equal the effective tax rate under mark-to-market and the statutory tax rate on these highest and lowest paths since the imputed return each year—which is a blended average of the actual returns—equals the actual return if the returns are constant. The ex post effective tax rate varies from the statutory rate under the retrospective method when returns are mixed.\textsuperscript{75} It is lower than the statutory rate if high returns precede low returns (for example, on the \textit{xy} path) and higher than the statutory rate if low returns precede high returns (for example, on the \textit{yx} path).

Table I illustrates these points by showing the end value and the ex post effective tax rate under the realization, expected return and retrospective methods on the four paths of a two-period investment. I assume an investment of $100 with a high return of 35\% (\textit{x} = 1.35) and a low return of 15\% (\textit{y} = .85). These are of equal probability. The average or expected rate of return is 10\%, which is also the imputation rate under the expected return method and the interest rate charged under the retrospective method. The tax rate is 40\%. Under the expected return method, I assume the tax in Period 1 is funded externally by borrowing at 10\%.

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\textsuperscript{74} What is deferred under the expected return method is the tax on gain in excess of the expected rate of return. Consider a two-period investment of an amount \(i\) that grows in value by a factor of \(x\) for two periods to reach an end value of \(ix^2\). A mark-to-market rule would impose a tax at a rate of \(t\) of \((ix - i)t\) in Period 1 and a tax of \((ix^2 - ix)t\) in Period 2. Assuming an expected rate of return of \(r\) expressed as a factor, the expected return method would impose a tax of \((ir - i)t\) in Period 1 and a tax of \((ix^2 - ir)t\) in Period 2. The expected return method defers \(i(x - r)t\) tax from Period 1 to Period 2.

\textsuperscript{75} For a two-period investment that changed in value by a factor of \(x\) in Period 1 and a factor of \(y\) in Period 2, a mark-to-market rule would impose a tax of \((ix - i)t\) in Period 1 and a tax of \((ixy - ix)t\) in Period 2. The imputation rate under the retrospective method expressed as a factor would be \((ixy/i)^{1/2}\). Under the retrospective method, a tax of \(((ixy/i)^{1/2} - i)t\) would be attributed to Period 1 and a tax of \((ixy - i(ixy/i)^{1/2})t\) would be imposed in Period 2. The retrospective method defers \((x - (ixy/i)^{1/2})t\) tax from Period 1 to Period 2.
Table 1
End value and effective tax rate on four paths of a two-period investment of $100

<table>
<thead>
<tr>
<th>path</th>
<th>pretax return</th>
<th>expected return</th>
<th>retrospective realization</th>
<th>expected return</th>
<th>retrospective realization</th>
<th>effective tax rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>xx</td>
<td>$182.25</td>
<td>$194.11</td>
<td>$149.35</td>
<td>36.83%</td>
<td>$148.51</td>
<td>37.54%</td>
</tr>
<tr>
<td>xy</td>
<td>114.75</td>
<td>108.61</td>
<td>108.68</td>
<td>108.85</td>
<td>40.74</td>
<td>40.32</td>
</tr>
<tr>
<td>yx</td>
<td>114.75</td>
<td>108.61</td>
<td>108.68</td>
<td>108.85</td>
<td>40.74</td>
<td>40.32</td>
</tr>
<tr>
<td>yy</td>
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<td>83.11</td>
<td>83.71</td>
<td>83.35</td>
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<td>43.27</td>
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<td>112.36</td>
<td>112.39</td>
<td>112.60</td>
<td>40.00</td>
<td>39.80</td>
</tr>
</tbody>
</table>

The difference in the ex post effective tax rate on the different outcomes may be worrisome for several reasons. One reason explored at length below is the possibility it creates for strategic trading. But strategic trading is not possible with the imaginary absolutely illiquid investment. For such an investment, the mistaxation of non-normal returns may be worrisome for two other reasons. The differences in the ex post effective tax rates may be troubling on equity grounds since people will pay tax at different rates depending on how their investments perform. This may not be a significant concern, however, if most investors hold portfolios with offsetting returns.

The other possible concern stems from the effect these ex post differences may have on ex ante valuation of investments. If people were risk neutral, they would value risky investments by discounting outcomes by their probability to determine the expected return. The effective tax rate on the investment would be a function of the effect of the tax on that average return. The bottom line on Table 1 shows the average return and the effective tax rate on that average return for the hypothetical two-period investment. Under the expected return

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66 See text accompanying notes 95-106.
67 Effective tax rate equals 100(1 − (after-tax rate of return/pretax rate of return)). Unlike Srnadd, Bonds, note 14; Constantindes, Capital Market, note 15; and Constantindes & Ingersoll, Bond Trading, note 15, I state the value of tax deferral and the timing option in terms of the effective tax rate, not the effect on asset price. There are several reasons for this. The most important is that results from the stock data model appear as dollar returns which (to permit comparison of investments of different lives) must be translated into rates of return. The effective tax rate on the investment would be a function of the effect of the tax on that average return. The effective tax rate ably reflects how tax affects rate of return. I thought it wise to express the results of the binomial distribution model in the same form as the stock data model to permit direct comparison. Second, the translation of a reduction in effective tax rate to an increase in asset value requires assumptions about the capitalization of tax preferences in asset prices that I prefer not to make. Third, my research objective is different from that of Constantindes, whose primary concern was accuracy in the use of observed bond prices to derive the term structure of interest. If long bonds trade at a premium because of the greater value of the timing option, then disregarding that premium overstates the slope in the yield curve. This requires analysis of the effect of the timing option on bond prices.
method, the low tax rate on the high yield outcome is precisely offset by an effective tax rate that is higher than the statutory rate on all other outcomes so the effective tax rate on the average return equals the statutory rate. The effective tax rate on the average return does not quite equal the statutory rate under the retrospective method.\footnote{Why this occurs can be shown with the two-period investment. The mark-to-market and the retrospective methods are equivalent on the \textit{xx} and \textit{yy} paths, that is, the paths with constant returns. The methods differ on the \textit{xy} and \textit{yx} paths. On both paths, the retrospective method imputes income in Period 1 by assuming the investment grew at a factor of \((ixyli)^{\text{ixy}}\) when the actual change in value is by a factor of \(x\) on the \textit{xy} path and a factor of \(y\) on the \textit{yx} path. These errors do not cancel out. The difference in the Period 1 taxes under the mark-to-market and the retrospective methods is \(i(x - (ixyli)^{\text{ixy}})t\) on the \textit{xy} path and \(i(y - (ixyli)^{\text{ixy}})t\) on the \textit{yx} path. If \(x>y\), the \textit{xy} differential is positive and the \textit{yx} differential is negative. The sum of these differentials is positive and increases with the gap between \(x\) and \(y\).} As will be shown shortly, this rate declines under the retrospective method as the volatility and term of an investment increases. The effective tax rate on the average return does not equal the statutory rate under the realization method either. The tax savings is due to deferral and is independent of the volatility of the return. The 24 cents average tax savings equals the tax savings from deferring tax at a 40% rate on $10 of income one year with an interest rate of 10%.

Chart 1 takes the two-period investment depicted in Table 1 and extends it through seven periods to show the effect of increasing the term of an investment on the effective tax rate on the average return. The effective tax rate under the realization method declines steeply as the term increases. The effective tax rate under the retrospective method declines gradually. The effective tax rate under the expected return method remains constant at the statutory rate of 40%.
Chart 1
Investment horizon and effective tax rate with no trading

Chart 1 assumes $x = 1.35$, $y = .85$ and $Pr(x) = Pr(y) = 50\%$.

Chart 2 holds the term of the investment at five periods and alters its volatility by increasing $x$ and decreasing $y$, which remain of equal probability. Volatility is measured by the standard deviation in the pretax rate of return in Period 5. The effective tax rate under the retrospective method declines steeply as volatility increases. The effective tax rates under the expected return and realization methods remain constant.
Mary Louise Fellows has suggested that the rate of interest charged on deferred taxes under the retrospective method might be set equal to the actual rate of return on an investment. This approach makes little sense for investments with returns that may vary significantly from the expected return because the actual return will bear no relation to the investor’s normal borrowing rate. Indeed, such an approach would have an horrendous effect on risky investments, as Chart 3 illustrates using a five-period investment. Charging interest on deferred taxes at the actual rate of return of an investment can result in an effective tax rate on the average rate of return in excess of 100%. What this means is that an investment with a positive pretax expected rate of return will have a negative after-tax expected rate of return.

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80 Fellows, note 5, at 748-51. Professor Fellows would use the investment’s rate of return only for computing interest on deferred taxes on gains; she would use the U.S. Treasury Note rate to compute interest paid on deferred tax refunds on losses.
Volatility and effective tax rate under retrospective method with interest charged at actual rate of return

Calculating interest at the actual rate of return overtaxes because the value to a taxpayer of tax deferral is his normal cost or return on capital and not the supranormal return he earns on a risky investment that pays off. Calculating interest on deferred taxes on subnormal but positive returns at lower than normal rates would not offset this effect because of the much larger tax liability on supranormal returns. Strategic trading partly alleviates this effect, but effective tax rates still can exceed 100% even with strategic trading.

---

81 I assume that the government would charge interest rather than pay interest on deferred refunds when the investment declines in value. This anomaly follows from applying the same formula for calculating interest on all outcomes. I do this for the sake of convenience. It has little effect on the overall result: Overtaxation persists albeit in smaller degree if interest is paid on deferred refunds. In the case used to generate Chart 3, the expected return under the retrospective method with interest charged at the actual rate of return is $118.71 (a 5.72% rate of return) if interest is paid on deferred refunds, and $117.64 (a 5.56% rate of return) if interest is charged.

This assumption is consistent with the assumption underlying charging interest at the actual rate of return: When an investment gains in value, deferred taxes are reinvested at the investment's rate of return. Following this logic, deferred refunds would have been reinvested and lost, and so withholding the refund saves the taxpayer money which the government might properly deduct from the eventual refund. There is a striking feature of calculating taxes on this assumption: On every outcome, the tax reduces the pretax rate of return by precisely the tax rate.
2. **Stock Model**

The stock model generally bears out the results of the binomial distribution model with additional wrinkles because of the presence of dividends (which reduces the element of deferral) and because of difficulty in specifying the imputation rate under the expected return method. I collected monthly price and all the distribution data on a sample of 500 stocks. $10,000 was invested in each stock that appeared at the beginning of a test period. This stock was held until the end of the period or until it ceased trading. Dividends and other nonterminal distributions were reinvested in the stock. I assumed a statutory tax rate of 40%. Stock was sold if necessary to obtain cash to pay tax assessed under the expected return and mark-to-market methods.

Table 2 shows the effective tax rate for stocks that were held for the entire test period. To determine the effective tax rate, I calculated the rate of return on the average end value for all stocks in that period under each tax rule and compared that to the rate of return on the average end value for all stocks in a tax-free environment. When the analysis includes all stocks—including those that ceased trading during the period—I calculated the rate of return on each stock under each tax rule and in a tax-free environment and compared the average of those rates of returns. Both methods are explained in the Appendix. Table 2 is based on data from six five-year periods (starting on January 1, 1960, 1965, 1970, 1975, 1980 and 1985), three 10-year periods (starting on January 1, 1960, 1970 and 1980) and two 15-year periods (starting on January 1, 1960 and January 1, 1975). The number of stocks is reported as $n$. The number of stocks exceeds the sample size (500) in the five-year set because some stocks appear in multiple periods.

### Table 2

<table>
<thead>
<tr>
<th>Investment Horizon</th>
<th>Realization</th>
<th>Expected Return</th>
<th>Retrospective</th>
<th>Mark-to-Market</th>
<th>$n$</th>
</tr>
</thead>
<tbody>
<tr>
<td>5-year</td>
<td>36.07</td>
<td>36.98</td>
<td>38.55</td>
<td>41.43</td>
<td>827</td>
</tr>
<tr>
<td>10-year</td>
<td>32.84</td>
<td>35.41</td>
<td>39.15</td>
<td>40.09</td>
<td>328</td>
</tr>
<tr>
<td>15-year</td>
<td>27.23</td>
<td>28.89</td>
<td>32.91</td>
<td>43.73</td>
<td>169</td>
</tr>
</tbody>
</table>

The effective tax rate under the realization method is higher than it was under the binomial distribution model probably due to the fact

---

82 The following table shows the effective tax rate based on the average rate of return. This table includes stocks that traded for less than the entire period.
that many stocks in the sample paid taxable dividends, which reduces the element of deferral. The effective rate under the realization method drops somewhat if one looks only at stocks that paid dividends at an annual rate of less than 2%—to 34.08% in the five-year set, 28.45% in the 10-year set and 19.85% in the 15-year set. These rates probably are increased by the unusually low rate of return on low dividend paying stocks. The effective tax rate under the realization method is inversely correlated with the rate of return.

The effective tax rate under the expected return method is well below the statutory tax rate and it declines as the investment horizon increases. This is not surprising. The imputation rate—I used the long-term bond rate—was well below average rates of return on stock, so there is a significant element of anticipated deferral under the expected return method. In the five-year set, the average rate of return on stocks held throughout the five-year period was 9.45%. The average rates of return in the 10-year and 15-year sets were

<table>
<thead>
<tr>
<th>Investment Horizon</th>
<th>Realization</th>
<th>Expected Return</th>
<th>Retrospective</th>
<th>Mark-to-Market</th>
<th>n</th>
</tr>
</thead>
<tbody>
<tr>
<td>5-year</td>
<td>31.81</td>
<td>32.73</td>
<td>32.95</td>
<td>36.13</td>
<td>1095</td>
</tr>
<tr>
<td>10-year</td>
<td>28.32</td>
<td>31.02</td>
<td>31.21</td>
<td>30.44</td>
<td>529</td>
</tr>
<tr>
<td>15-year</td>
<td>23.87</td>
<td>24.60</td>
<td>27.34</td>
<td>29.64</td>
<td>169</td>
</tr>
</tbody>
</table>

Average annual dividend rates for stocks in the set by decade and over the entire period were:

- Overall: 4.04
- 1960-1970: 3.32
- 1980-1990: 4.45

The average dividend rate includes the 1,095 stocks in the sample that appeared at the beginning of any five-year set. The dividend rate is the total dividends paid during the year divided by the share price on the first trading day of the year. Only distributions classified as dividends in the CRSP database were so treated.

There were 204 stocks in the five-year set, 66 in the 10-year set and 28 in the 15-year set.

Stocks paying dividends at a rate lower than 2% had an average rate of return of 6.55% in the five-year and 10-year sets and 7.79% in the 15-year set. This compares to rates of return of 9.45%, 9.82% and 9.72% on all stocks in the sample. Low dividend paying stocks also had higher rates of volatility. The average volatility of the monthly rate of return on all 1,095 stocks in the 5-year set was 10.16. For the 309 stocks that paid dividends at an average rate of less than 2%, the average volatility was 13.54. For the 123 stocks that never paid dividends, the average volatility was 16.1.

For years after 1984, I used the long-term AFR under § 1274(d). For 1970 to 1984, I used the 10-year Treasury bond rate. For 1960 to 1969, I used the rate for three- to five-year government securities. Sources are provided in the Appendix, at note 116.

This is the average of the rates of return in a tax-free environment on the 827 stocks in the sample that appeared at the beginning of any five-year set and that traded throughout the entire five years.
9.82% and 9.72%. The average long-bond rate over the entire 30-year period was 8.69%. The long-bond rate was closest to the average rate of return on stocks in the period from 1980 to 1985 (the average rate of return on stocks was 12.62% while the average long-bond rate was 12.21%). During that period, the effective tax rate under the expected return method rises to 37.9%.

One response to the persistent gap between the average rate of return on stocks and the bond rate is to increase the current bond rate by some factor to determine the expected return on stock. Table 3 shows the effect of increasing the imputation rate under the expected return method by adding 2% to 6% to the long-bond rate. As would be expected, the effective tax rate rises with the imputation rate, though it never reaches the statutory rate of 40%. The table includes stocks that traded for the entire period and shows the effective tax rate on the rate of return on the average end value of stocks in the set.

Table 3
Effective tax rate under the expected return method with an adjustment in the long-bond rate

<table>
<thead>
<tr>
<th>Adjustment in long-bond rate</th>
<th>5-year set</th>
<th>10-year set</th>
<th>15-year set</th>
</tr>
</thead>
<tbody>
<tr>
<td>0%</td>
<td>36.98</td>
<td>35.41</td>
<td>28.89</td>
</tr>
<tr>
<td>2%</td>
<td>37.51</td>
<td>36.72</td>
<td>29.73</td>
</tr>
<tr>
<td>4%</td>
<td>38.05</td>
<td>38.24</td>
<td>30.83</td>
</tr>
<tr>
<td>6%</td>
<td>38.62</td>
<td>39.65</td>
<td>32.05</td>
</tr>
</tbody>
</table>

The greatest effect is in the 10-year set, where a 2% increase in the imputation rate increases the effective tax rate by slightly less than 1.5%.

The stock data supports the hypothesis that the effective tax rate declines under the retrospective method as the term of an investment increases. Table 2 shows that the effective tax rate under the retrospective method decreases from 38.5% in the five-year set to 32.9% in the 15-year set. The small size of the 15-year set and the fact that the

---

88 This is the average of the rates of return in a tax-free environment on 328 stocks in the 10-year sets and 169 stocks in the 15-year sets. This includes stocks that appeared at the beginning of any set and that traded throughout the entire period.

89 See note 86.

90 The stock figure is based on 152 stocks that traded for the entire period from January 1, 1980 to December 31, 1984. It is the average of the rates of returns on individual stocks. For the source of the bond figure, see note 86.
effective tax is higher in the 10-year set than in the five-year set, however, caution against drawing too strong an inference on this point.\textsuperscript{91}

The effective tax rate decreases as volatility increases under the retrospective method, as it did in the binomial distribution model. Chart 4 demonstrates this point with data from the five-year set. The chart groups stocks by their volatility and calculates the effective tax rate on the rate of return on the average end value of stocks in each group. Volatility is measured by the standard deviation in the monthly rate of return on the stock, including price changes and distributions. The number of stocks at each level of volatility is reported in parentheses.

\textbf{Chart 4}

\textit{Volatility and the effective tax rate under the retrospective method without trading}\textsuperscript{92}

At levels of volatility higher than those shown on the chart, the effective tax rate fluctuates wildly, which I believe is due to the fact that rates of return on the average end values were near zero at higher

\textsuperscript{91} The increase in the 10-year set largely is attributable to the 1970 investment set. The effective tax rate for that set under the retrospective method was an extraordinary 39.04% without trading (it was 24.99% with trading). This was higher than the effective tax rate under the mark-to-market method, which was 36.09% without trading. This result probably is due to the path the market took in the 1970's. It dove steeply in 1973 and 1974 and then climbed so there was a net gain by the end of the decade. Board of Governors of the Federal Reserve System, Annual Statistical Digest: 1970-1979, at 178 tbl. 23A (1981) [hereinafter Federal Reserve Statistics]. Under the retrospective method, where the stock was held throughout the decade, this late gain was attributed back to early years.

\textsuperscript{92} This includes 827 stocks that traded at the start of each five-year set and traded for the entire set. Less than 827 stocks are shown in Chart 4 because a few stocks with very high levels of volatility are omitted. See note 85.
levels of volatility. The effective tax rate is not meaningful at near-zero rates of return. The uptick in effective tax rates at the right side of Chart 4 may be due to a decrease in the rate of return on average end value at those levels of volatility (the effective tax rate under the retrospective method moves inversely with the rate of return).

B. Costless Trading

Where an investment can be traded at zero cost, strategic trading significantly diminishes the effective tax rate on investments with volatile prices under the realization, expected return and retrospective methods. Indeed, effective tax rates converge under the three methods at extremely high levels of volatility.

A sale produces a tax benefit under the realization and expected return methods when the price of an asset drops below its adjusted basis. The adjusted basis of a stock generally is its cost, though some events (for example, nontaxable distributions) may reduce or require the apportionment of basis. Under the expected return method, the adjusted basis of an asset

93 The following table shows the effective tax rate at levels of volatility higher than reported on Chart 4 and the number of stocks(n) in the sample at that rate.

<table>
<thead>
<tr>
<th>Volatility</th>
<th>Effective Tax Rate</th>
<th>n</th>
</tr>
</thead>
<tbody>
<tr>
<td>14 to 17</td>
<td>46.03</td>
<td>52</td>
</tr>
<tr>
<td>17 to 22</td>
<td>34.91</td>
<td>26</td>
</tr>
<tr>
<td>over 22</td>
<td>51.71</td>
<td>4</td>
</tr>
</tbody>
</table>

Rate of return on the average end value in the group by volatility is as follows:

This includes 827 stocks that traded at the start of each five-year set and traded for the entire set. The stocks are grouped by the average volatility in their monthly rate of return. The number of stocks in each group is in the parentheses. Rate of return, the vertical axis, is calculated on the average end value of all stocks in the group.
would be increased by income imputed on the asset. Under the retrospective method, a sale may produce a tax benefit after a period of subnormal returns because a sale prevents blending of expected normal future returns with those low returns, resulting in the dilution and partial deferral of the low returns.

1. **Binomial Distribution Model**

To understand the tax incentive to sell an asset with a volatile price after a loss in value, consider the position of an investor who has experienced a loss and who expects the asset to either increase or decrease in value in the following period or periods. The simplest case is where the asset will be held for at most one additional period. Consider a two-period investment of $100 in an asset that may increase in value by a factor of 1.35 or decrease in value by a factor of .85 each period. In Period 1, the asset decreases in value to $85. The investor may sell the asset, realize the loss, and reinvest the proceeds in a one period investment with the same possible returns. Or she may hold on to the asset to see if it regains its value or continues to decline. Table 4 shows the investor's expected after-tax rate of return on the $85 value of her investment at the end of Period 1 for each choice.

<table>
<thead>
<tr>
<th></th>
<th>Hold</th>
<th>Sell</th>
<th>% Improvement by Selling</th>
</tr>
</thead>
<tbody>
<tr>
<td>Realization</td>
<td>5.6%</td>
<td>6%</td>
<td>7.14%</td>
</tr>
<tr>
<td>Expected Return</td>
<td>5.34%</td>
<td>6%</td>
<td>12.36%</td>
</tr>
<tr>
<td>Retrospective</td>
<td>5.71%</td>
<td>6%</td>
<td>5.08%</td>
</tr>
</tbody>
</table>

Holding produces below normal after-tax returns (6% is the assumed normal return) in Period 2 under the first two methods because it defers realization of the Period 1 loss. Holding produces a below normal return under the retrospective method because realization of the Period 1 loss may be partially deferred if there is a gain in Period 2. The loss is realized on a sale in Period 1, providing a normal rate of return in Period 2.95

---

95 The investor also would sell after a loss if the initial investment horizon was more than two periods. In the preceding example, if the investor had an initial investment horizon of three years, she would compare the expected return if she sold after Period 1 with the expected return if she waited to sell after another loss in Period 2. Selling remains advantageous despite the lengthening of the investment horizon, though less so. The return is on the $85 value of the investment after the loss.
The timing option increases the average after-tax return on an investment and so decreases the effective tax rate that an investment is expected to bear. Chart 5 shows that the effective tax rate on the average rate of return on an investment drops rapidly under all three tax rules when the volatility of an investment increases. It assumes a five-period investment. The volatility of the investment is altered by varying the high and low return while keeping their probability equal. I assume an average return of 10% and a tax rate of 40%.

<table>
<thead>
<tr>
<th></th>
<th>Sell in Period 1</th>
<th>Hold and sell if loss in Period 2</th>
<th>% Improvement by selling in Period 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Realization</td>
<td>6.2%</td>
<td>5.89%</td>
<td>5.26%</td>
</tr>
<tr>
<td>Expected Return</td>
<td>6.14%</td>
<td>5.63%</td>
<td>9.06%</td>
</tr>
<tr>
<td>Retrospective</td>
<td>6.08%</td>
<td>5.84%</td>
<td>4.11%</td>
</tr>
</tbody>
</table>

A sale in Period 1 produces a higher than normal after-tax return because funds are reinvested in a similar two-period investment with similar tax benefits.

Extending the investment horizon diminishes the gain from selling after a loss in Period 1, but the gain remains significant. The chart below illustrates using an investment with the same attributes as above \((x = 1.35, y = .85, \Pr(x) = \Pr(y) = 50\%)\). It shows the percent improvement in expected after-tax return on the $85 value of the investment at the end of Period 1 from selling after a loss in Period 1 for investments of varying original terms.

![Investment horizon and value of sale after loss](chart.png)

The percentage improvement in the subsequent after-tax rate of return from selling after a Period 1 loss decreases as the investment horizon lengthens, but at a diminishing rate. This pattern holds for most investments, though different distributions of returns produce differently positioned and shaped curves. The one exception is where there is a high probability of a low gain and a low probability of a large loss \((x \text{ is near } 1.1 \text{ and } y \text{ is near } .1)\), where increasing the term of investment increases the benefit of a Period 1 sale after a loss under the expected return method and the retrospective method.
Volatility and effective tax rate with costless trading

Effective tax rates also decrease as the investment horizon lengthens. Chart 6 shows the effective tax for investments with lives of two to seven years with constant $x$ and $y$.

$96 \ x = 1.3, \ y = .9, \text{ and } Pr(x) = Pr(y) = .5.$
The effective tax rate under the three methods is sensitive not just to the volatility of price, as expressed by the standard deviation in an investment’s possible returns, but also to the distribution of returns. For the investment depicted in Charts 5 and 6, returns are normally distributed around the average return \( r \). Table 5 shows a series of investments with a similar standard deviation in returns (approximately 20)\(^{97}\) but with returns differently distributed around \( r \). For example, the first line is an investment with a very high probability of a return slightly above the normal return of 10\% and a small probability of a total loss.

Table 5

<table>
<thead>
<tr>
<th>( x - 1 )</th>
<th>( y + 1 )</th>
<th>( \Pr(x) )</th>
<th>( \Pr(y) )</th>
<th>Standard Deviation</th>
<th>Realization</th>
<th>Expected Return</th>
<th>Retrospective</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.107415</td>
<td>-1.0000</td>
<td>0.993304</td>
<td>0.006696</td>
<td>20.00593</td>
<td>35.3</td>
<td>39.7</td>
<td>39.7</td>
</tr>
<tr>
<td>0.20974</td>
<td>-0.7000</td>
<td>0.879372</td>
<td>0.120628</td>
<td>20.00955</td>
<td>32.7</td>
<td>36.0</td>
<td>37.4</td>
</tr>
<tr>
<td>0.3155</td>
<td>-0.5000</td>
<td>0.735745</td>
<td>0.264255</td>
<td>20.00713</td>
<td>31.6</td>
<td>34.0</td>
<td>36.2</td>
</tr>
<tr>
<td>0.571</td>
<td>-0.3000</td>
<td>0.459242</td>
<td>0.540758</td>
<td>20.00105</td>
<td>31.2</td>
<td>33.1</td>
<td>35.7</td>
</tr>
<tr>
<td>0.9</td>
<td>-0.2037</td>
<td>0.275165</td>
<td>0.724835</td>
<td>20.00774</td>
<td>31.3</td>
<td>32.7</td>
<td>35.4</td>
</tr>
<tr>
<td>3</td>
<td>-0.0825</td>
<td>0.059205</td>
<td>0.940795</td>
<td>20.00525</td>
<td>32.7</td>
<td>33.2</td>
<td>35.3</td>
</tr>
<tr>
<td>6</td>
<td>-0.0582</td>
<td>0.026113</td>
<td>0.973887</td>
<td>20.00303</td>
<td>33.4</td>
<td>33.6</td>
<td>35.3</td>
</tr>
</tbody>
</table>

Increases in \( x \) moving down the table require decreases in \( y \) if the standard deviation in the returns is held constant, which results in decreases in \( \Pr(x) \) and increases in \( \Pr(y) \). Effective tax rates decrease and then increase moving down the table. Intuitively, the timing option is valuable under these rules because of the value of tax deferral on supranormal returns. Moving down the table at first increases the value of the timing option as \( x \) increases, but eventually the value of the timing option decreases because the increasing probability of a loss diminishes the expected period of deferral since a loss will trigger a sale. Effective tax rates constantly decrease moving down the table under the retrospective method because the method undertaxes investments when losses follow gains by blending the loss with the earlier gain. The increased probability of loss moving down the table increases this effect.\(^{98}\)

---

97 The standard deviation in the return is a poor measure of the volatility of the investments at the top and bottom of the table because the returns are widely skewed. The most likely outcomes are near normal, but there are very low probability outcomes that are far from normal. For example, in the investment in the top line, there is a 96.7\% chance that the investment will return 10.7\% over five years and a 3.3\% chance that it will become almost valueless.

98 Under the retrospective method, the improvement in return is not solely a function of the timing option. Returns increase moving down the table under the retrospective...
2. *Stock Model*

Effective tax rates under the realization and retrospective methods also are close and far below the stated tax rate in the stock model. I analyzed the effect of strategic trading on the returns for investments of 5-, 10- and 15-year lives. An investment of $10,000 was made in each stock that appeared at the beginning of a test period. Dividends and other non-terminal distributions were reinvested in a stock. The stock price was checked on the last trading day of the month and the stock was sold if the price was less than the stock's basis. The proceeds from sale plus any tax refunds were immediately reinvested in the stock.

Table 6 includes stocks that traded for the entire period and shows the effective tax rate based on the rate of return on the average end value of all stocks in the set.

<table>
<thead>
<tr>
<th>Investment Horizon</th>
<th>Realization Return</th>
<th>Expected Return</th>
<th>Retrospective</th>
<th>Mark-to-Market</th>
<th>n</th>
</tr>
</thead>
<tbody>
<tr>
<td>5-year</td>
<td>33.00</td>
<td>33.59</td>
<td>35.81</td>
<td>39.44</td>
<td>827</td>
</tr>
<tr>
<td>10-year</td>
<td>24.85</td>
<td>25.96</td>
<td>31.32</td>
<td>37.48</td>
<td>328</td>
</tr>
<tr>
<td>15-year</td>
<td>24.25</td>
<td>25.25</td>
<td>30.21</td>
<td>41.99</td>
<td>169</td>
</tr>
</tbody>
</table>

The effective tax rate under the retrospective method is midway between those under the realization and mark-to-market methods. Effective tax rates drop significantly under all three methods when the investment horizon increases from five to 10 years, but drop slightly when the investment horizon increases from 10 to 15 years.

---

99 Table 6 includes all stocks in the sample of 500 that traded at the beginning of any 5-, 10- or 15-year set until the end of the period from 1960 to 1990. The five-year sets began every five years from January 1, 1960, the 10-year sets every 10 years starting on that date, and the 15-year sets every 15 years. \( n \) is the number of stocks. The following table shows the effective tax rate on the average rate of return. It includes stocks that traded for less than the entire period.

<table>
<thead>
<tr>
<th>Investment Horizon</th>
<th>Realization</th>
<th>Expected Return</th>
<th>Retrospective</th>
<th>Mark-to-Market</th>
<th>n</th>
</tr>
</thead>
<tbody>
<tr>
<td>5-year</td>
<td>29.65</td>
<td>30.09</td>
<td>31.37</td>
<td>34.10</td>
<td>1095</td>
</tr>
<tr>
<td>10-year</td>
<td>19.37</td>
<td>20.09</td>
<td>24.01</td>
<td>27.93</td>
<td>450</td>
</tr>
</tbody>
</table>
One can get a rough sense of the relative value of deferral and the timing option at different investment horizons by comparing effective tax rates at varying lives with and without trading. Chart 7 combines Tables 2 and 6.

**Chart 7**
A comparison of effective tax rates with and without trading

The combination of increasing the investment horizon from five to 10 years and adding the timing option steeply reduces the effective tax rate. With trading, lengthening the investment horizon from 10 to 15 years causes little further reduction in the effective tax rate, particularly under the realization and expected return methods. At the same time, the greatest part of the reduction in the effective tax rate with a 15-year investment horizon is due to deferral and does not depend upon the timing option.

The assumption of monthly trading may be unrealistic for some investors who do not consider tax reasons for selling until the end of the tax year. To see how an investor who made tax-motivated trades only at year end would fare, I analyzed returns with trading opportunities limited to the last trading day of the year. Table 7 shows the results.
Table 7

Effective tax rate with costless and annual trading

<table>
<thead>
<tr>
<th>Investment Horizon</th>
<th>Realization</th>
<th>Expected Return</th>
<th>Retrospective</th>
<th>Mark-to-Market</th>
<th>n</th>
</tr>
</thead>
<tbody>
<tr>
<td>5-year</td>
<td>34.99</td>
<td>35.71</td>
<td>37.76</td>
<td>41.16</td>
<td>827</td>
</tr>
<tr>
<td>10-year</td>
<td>28.17</td>
<td>29.64</td>
<td>34.37</td>
<td>39.88</td>
<td>328</td>
</tr>
<tr>
<td>15-year</td>
<td>25.19</td>
<td>26.36</td>
<td>31.19</td>
<td>43.59</td>
<td>169</td>
</tr>
</tbody>
</table>

Although the effective tax rates shown in Table 7 are higher than those in Table 5 (monthly trading), especially in the 10-year set, the timing option continues to have significant value, particularly at a 10-year investment horizon. (The effective tax rate increases under the mark-to-market method with annual rather than monthly trading because losses can be accelerated within the year with monthly trading.) Effective tax rates under all of the methods vary across periods. This is not surprising because the ex post effective tax rate on an investment is affected by the rate of return on the investment and the path the investment’s price takes. With a large number of stocks over several sample periods, rate and path dependent effects tend to average out so that the ex post effective rate approximates the effective tax rate that an investment can be expected to bear on average. But in a single period, broad movements in the market may cause certain paths or results to dominate so the ex post rate varies from the expected rate. These variations across periods, however, caution against assuming that the ex post effective tax rates found in the stock data model correspond to the average effective tax rate facing an investor, particularly in the 10- and 15-year data sets, where there were few test periods.

Tables 8-10 show the effective tax rates by period for each investment set.

---

100 Table 7 includes all stocks in the sample of 500 that traded at the beginning of any 5-, 10- or 15-year set until the end of the period from 1960 to 1990. The five-year sets began every five years from January 1, 1960, the 10-year sets every 10 years starting on that date, and the 15-year sets every 15 years. n is the number of stocks.
The period 1970 to 1975 stands out in the five-year set: The effective tax rate drops steeply under the retrospective method and soars under the mark-to-market method. This period was atypical in the stock market. The worst drop in the market from 1960 to 1990 was in 1973 and 1974.  

The Standard and Poor Composite Index dropped from 118.42 on January 1, 1973 to 72.56 on January 1, 1975. The Dow Jones Index dropped from 64.38 to 38.56 over the same period. Federal Reserve Statistics, note 91, at 180-81 tbl. 23A.
this method, and the high effective tax rate under the mark-to-market method means that they would fare the best.\textsuperscript{102}

The effective tax rates on investments in stock decline as the volatility in the monthly rate of return of the stock increases under all of the methods. The relationship between volatility and effective tax rate is very pronounced in the 10-year set, as Chart 8 shows by dividing stocks into groups based on their volatility.

\textsuperscript{102} This occurs because taxes were funded internally by selling stock. An investor might do better under mark-to-market than the other methods when a stock rises in value and then drops. Under the mark-to-market method, the investor pays the tax on the rise in value by selling appreciated shares and then gets the benefit of a tax loss when the shares drop in value. The tax on the gain would seemingly offset the value of the refund on the loss (with the additional timing cost), but the tax is paid out of the disappearing appreciation in the value of the shares. Under the other methods, the investor holds the stock through the rise in value and gets no benefit from the lost appreciation.

For example, A buys 10 shares of stock at $100 per share. The stock doubles in value to $200 and he incurs a $400 tax liability ($1,000 gain x 40% tax rate). A sells two shares to pay the tax. Later the stock declines in value to $100. A's basis in the eight remaining shares is $1,600. He sells the shares and realizes an $800 loss for which he receives a tax refund of $320. A ends up with $1,120 ($800 realized on the eight shares plus a $320 refund). Had A not sold shares to pay tax on the disappearing gain, he would be left with $1,000.

The government paid the least in refunds to the hypothetical investor, by a significant margin, under the mark-to-market method. I maintained an account of tax paid for each stock, crediting it with interest at the short-term government bond rate. In the period 1970 to 1975, these accounts were negative, reflecting that more was paid out in refunds than was collected in taxes. Less was paid out, on average, under the mark-to-market method—$288 compared to $793 under the realization rule, $735 under the expected return method and $760 under the retrospective method. There is a loser: the person who buys the stock the investor sells to pay the tax on the appreciation. The government will share in this loss eventually.
The effective tax rate under the retrospective method goes off the chart to 52% in the group of six stocks with volatilities in the range of 13 to 14 due to one aberrant stock that had a rate of return in a tax-free environment in excess of 50%. If that stock is eliminated, the effective tax rate on the average rate of return on the other five stocks under the retrospective method is a more normal 22.26%. No significance should be accorded to the rise in effective tax rates at the right side of the chart. Few stocks appear in this range. At the far right, the effective tax rates drop off the chart, but these rates are meaningless because they are computed on a near-zero rate of return on average end value. The decline in effective tax rates cannot be attributed to factors other than the increase in volatility. If average rates of return increased with volatility, that would cause a decrease in the effective tax rate under all the methods, but the return does not consistently increase with volatility. The relationship of the effective tax rate to volatility is similar in the five-year and 15-year sets—a gradual decline until high levels of volatility, where the effective tax rate begins to rise and finally swings wildly.

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103 This includes 328 stocks that traded at the start of each 10-year set and traded for the entire set. The stocks are grouped by the average volatility in their monthly rate of return. The number of stocks in each group are in the parentheses. Rate of return, the vertical axis, is calculated on the average end value of all stocks in the group.

104 The rates are -86.57, -87.53 and -72.49.

105 The rate of return on the tax-free average end value is 2.94.

106 The returns across the volatility ranges are as follows:
C. Transaction Costs

Transaction costs include broker's fees, transfer taxes, spreads between offer and asking prices by market makers or brokers and, for assets with thin markets, sacrifices made in sales price by limiting the search for potential buyers. Rising trading costs quickly suppress strategic trading under all methods for investments of short lives. For longer-lived investments, strategic trading persists at moderate levels of trading costs under the realization and expected return methods while strategic trading is suppressed under the retrospective method.

1. Binomial Distribution Model

To see how a rise in trading costs can deter sales, consider a simple case: a two-period investment that experiences a loss at Period 1 where the return from Period 1 to Period 2 is risk-free. Under the realization or expected return methods, the benefit from selling at Period 1 is accelerating the loss by one period, or the interest that can be earned for one period on the tax refund from the loss.\(^\text{107}\) For investments with short horizons, modest levels of transaction costs can outweigh the benefit of sales even with large losses because the interest

\[\text{Rate of return} = \text{Volatility (standard deviation in monthly rate of return)}\]

\[\text{Assume } i \text{ is the investment, } y \text{ is the Period 1 return expressed as a factor (it is less than 1), } tc \text{ is the trading cost (a percentage of asset value), } r \text{ is the risk-free rate of return expressed as a factor, } t \text{ is the tax rate, and there is no trading cost in Period 2. A sale after Period 1 has a positive return if } [iy(1-tc) - iy(1-tc) - i]t(1 + (r - 1)(1 - t)) > iyr - (iyr - i).\]

Introducing risk does not change the analysis. The same function if the return in Period 2 may be \(x\) or \(y\) expressed as a factor with a probability of \(Pr(x)\) and \(Pr(y)\) is \([iy(1-tc) - iy(1-tc) - i]t(1 - x - 1)(1 - y)Pr(x) + iy(1-tc) - iy(1-tc) - i]t(1 + (y - 1)(1 - i)]Pr(y) > [iyx-(iyx-i)]Pr(x) + iy^2 - (iy^2 - i)]Pr(y). \text{ If } xPr(x) + yPr(y) = r, \text{ this function reduces to the same function as above.}\]
factor necessarily is small in proportion to asset value. In this simple example, if the loss at Period 1 is 30% of investment value, the interest rate is 10% and the tax rate is 40%, a sale at Period 1 is profitable under the realization method only if the cost of sale is less than 1.62% of asset value. Strategic sales can be profitable for investments with longer horizons on losses incurred at the beginning of their lives even at relatively high sales costs. In the example, if the life of the investment is four periods rather than two, a sale at Period 1 after a 30% loss is profitable up to a cost up to 10.88% of asset value, but the costs still erode the benefit gained from strategic trading.

Previously, I demonstrated that strategic trading produces a smaller tax benefit under the retrospective method than under the expected return method, but that the retrospective method is non-neutral with respect to risk if sales are impossible, while the expected return method is perfectly neutral. Thus, rising sales costs would be expected to suppress sales first under the retrospective method, but once sales costs become sufficient to suppress sales under the expected return method, that method will be neutral while the retrospective method will not. Chart 9, which shows the effective tax rate with rising sale costs, confirms this. It is based on a five-period investment with returns having a standard deviation of approximately 20%. Sales costs are defined as a percentage of the value of the investment.108 The timing option ceases to affect return when the line flattens, that is, at the level of sales costs where strategic sales no longer are profitable.

108 In order that the expected rate of return remain \( r \), in this example 10%, \( x \) and \( y \) are adjusted notwithstanding the expected sale in the last period. I also decreased the variance in the return with the increase in transaction costs to keep volatility constant. Holding the variance in the return constant while increasing transaction costs greatly increases the volatility of returns of an investment at high levels of cost.
Chart 9
Effective tax rates with rising trading costs

Chart 10 shows how increasing volatility affects the effective tax rate with constant sale costs at 10% of asset value. It also assumes a five-period investment.

Chart 10
Volatility and effective tax rate at 10% trading costs

The drop in effective tax rate under the realization and the expected return methods is purely a function of the timing option. The drop in
effective tax rate under the retrospective method is independent of the timing option for most of the range, as Chart 11 illustrates.

Chart 11

Effective tax rates under the retrospective method with free, costly and no trading

Chart 11 shows the effective tax rates for a five-period investment under the retrospective method with free trading, no trading and costly trading (10% of asset value) at increasing levels of volatility. The timing option would become valuable when the effective tax rates between no trading and costly trading diverge, which would occur only at extremely high levels of volatility.

2. Stock Model

The stock model bears out the persistence of strategic trading at moderate levels of trading costs under the realization and expected return methods. It also offers a rough measure of the magnitude of costs associated with tax-motivated trading. I tested at trading costs ranging from .25% to 5% of share price. This cost is assessed both on purchase and sale of a stock, so the cumulative cost of a sale to realize a tax loss with a repurchase is double the stated cost. To determine whether a sale was desirable, I assumed the stock’s price would remain unchanged for the remainder of the investment’s life and asked if the gain from accelerating recognition of that loss exceeded the af-
The binomial distribution models shows that the possibility of future changes in price need not be taken into account in deciding whether to sell after a loss under the realization and expected return methods if the future expected return is normal, and the stock data model seems to bear that out since this strategy consistently outperformed other strategies. However, I could not devise a successful test for making strategic sales under the retrospective method. Thus, the stock data results omit that method.

Tables 11-13 show the effective tax rate, average end value and the average trading cost of investments in stock that traded for the entire period.111 The effective tax rate is based on the rate of return on the

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109 The test for sale was $IF(((basis - share price) \times tax rate) \times (1 + interest rate (1 - tax rate)))^{(end date - cal date)/365} - ((basis - share price) \times tax rate > share price)(sell fee + buy fee) \times (1 - tax rate))$. "Sell fee" and "buy fee" are the cost in percent of selling and repurchasing the stock. "End date" is the expected terminal date of the investment. "Cal date" is the current date when the new share price is available.

110 Logically, a test for sales under the retrospective method must compare the cost of sale with the risk that a loss will be blended in with a future gain, and so partially deferred. Sale cost, magnitude of loss and the probability distribution of future prices all would be factors in the test. I tried several tests built on these elements, but none performed consistently better than the others at all levels of sale costs and investment lives. At sale costs of 2.5% or above, none of the tests performed appreciably better than did a hold strategy. At sale costs of .5% and below, none of the tests performed appreciably better than the simpler test of selling whenever there is a tax loss.

111 Excluding stocks that cease trading prior to the end of the period would understate the effective tax rate under the realization and expected return methods because premature terminations would eliminate deferral. A rise in trading costs might be expected to have a greater impact on the effective tax rate when stocks that prematurely terminate are added to the set because trades are made on the assumption that a stock will be held for the entire period. When a stock prematurely terminates trading, the costs of strategic trades are likely to turn out to have been incurred unwisely, particularly if the termination closely follows the trade.

It is difficult to assess the impact of strategic trading on returns on stocks that prematurely terminate trading. First, as explained in the Appendix, investments that terminate at different dates must be analyzed by computing the rate of return on each stock under various tax regimes, and then determining the effective tax rate on the average of those rates of return. The effective tax rate computed in this manner will be below the nominal tax rate even if the tax satisfies the criteria for perfect neutrality. Second, stocks that prematurely terminate trading tend to have much higher rates of return than do stocks that trade for the entire period. In the five-year set, for example, the rate of return on the 827 stocks that traded for the entire period was 9.4% while the rate of return on the 257 stocks that ceased trading prior to the end of the period was 26.3% in a tax-free environment. Thus, outcome-dependent effects on effective tax rates will muddle comparisons of the sets that include and exclude prematurely terminating stocks. Third, returns on stocks that prematurely terminate trading are more sensitive to a rise in trading costs even without strategic trading because there is less time to amortize the initial cost of purchasing stock. For example, in the five-year set, there were 41 stocks (out of 1,095) that ceased trading within one year. The tax-free rate of return on these stocks was 53% at zero trading costs and 28.4% at 5% trading costs, a reduction of almost 50%. For the 257 stocks that traded less than five years, the tax-free rate of return drops from 26.3% to 19.6%, approximately a 30% reduction. For stocks held for the entire period, the tax-free rate of return drops from
average end value for all stocks in the set. Trading costs were calculated for each stock by maintaining an account of costs as they were incurred. Amounts in this account were compounded at the current rate for low-risk corporate bonds.

9.4% to 7.1%, a 20% reduction. The more pronounced decrease in the rate of return when stocks that prematurely terminate trading are added to the set will tend to increase the effective tax rate as trading costs rise, even without the effects of more costly strategic trading. Finally, the 10- and 15-year sets are incomplete because they include only stocks that traded for at least five years in the 10-year set and 10 years in the 15-year set.

Subject to these caveats as to the meaningfulness of the data, I report the effective tax rate on the average rate of return for all stocks in the 5-, 10- and 15-year sets at rising trading costs below:

<table>
<thead>
<tr>
<th>Years</th>
<th>Cost</th>
<th>Realization</th>
<th>Expected Return</th>
<th>Mark-to-Market</th>
</tr>
</thead>
<tbody>
<tr>
<td>5 years</td>
<td>0</td>
<td>29.6</td>
<td>30.1</td>
<td>34.1</td>
</tr>
<tr>
<td></td>
<td>.25%</td>
<td>31.0</td>
<td>31.3</td>
<td>35.6</td>
</tr>
<tr>
<td></td>
<td>.50%</td>
<td>31.5</td>
<td>32.0</td>
<td>35.8</td>
</tr>
<tr>
<td></td>
<td>2.50%</td>
<td>32.4</td>
<td>33.3</td>
<td>35.2</td>
</tr>
<tr>
<td></td>
<td>5.00%</td>
<td>31.1</td>
<td>32.4</td>
<td>33.7</td>
</tr>
<tr>
<td>10 years</td>
<td>0</td>
<td>19.4</td>
<td>20.1</td>
<td>27.9</td>
</tr>
<tr>
<td></td>
<td>.25%</td>
<td>21.2</td>
<td>22.1</td>
<td>29.9</td>
</tr>
<tr>
<td></td>
<td>.50%</td>
<td>22.4</td>
<td>23.5</td>
<td>30.3</td>
</tr>
<tr>
<td></td>
<td>2.50%</td>
<td>26.9</td>
<td>28.8</td>
<td>30.9</td>
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<tr>
<td></td>
<td>5.00%</td>
<td>29.5</td>
<td>32.1</td>
<td>31.5</td>
</tr>
<tr>
<td>15 years</td>
<td>0</td>
<td>21.0</td>
<td>21.4</td>
<td>29.5</td>
</tr>
<tr>
<td></td>
<td>.25%</td>
<td>21.9</td>
<td>22.5</td>
<td>31.1</td>
</tr>
<tr>
<td></td>
<td>.50%</td>
<td>22.6</td>
<td>23.3</td>
<td>31.0</td>
</tr>
<tr>
<td></td>
<td>2.50%</td>
<td>24.9</td>
<td>25.8</td>
<td>31.3</td>
</tr>
<tr>
<td></td>
<td>5.00%</td>
<td>26.2</td>
<td>27.4</td>
<td>31.7</td>
</tr>
</tbody>
</table>
Table 11: Five-Year Set  
(n=827)

<table>
<thead>
<tr>
<th>Trading Cost</th>
<th>Tax-Free Trading</th>
<th>Realization Trading</th>
<th>Expected Return Trading</th>
<th>Mark-to-Market Trading</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>End Value</td>
<td>End Value</td>
<td>End Value</td>
<td>End Value</td>
</tr>
<tr>
<td>0%</td>
<td>19561</td>
<td>0</td>
<td>33.00</td>
<td>0</td>
</tr>
<tr>
<td>0.25%</td>
<td>19454</td>
<td>91.65</td>
<td>34.03</td>
<td>15665</td>
</tr>
<tr>
<td>5%</td>
<td>19347</td>
<td>182.74</td>
<td>35.02</td>
<td>370.26</td>
</tr>
<tr>
<td>2.5%</td>
<td>18523</td>
<td>891.99</td>
<td>37.41</td>
<td>14838</td>
</tr>
<tr>
<td>5%</td>
<td>17545</td>
<td>1732.63</td>
<td>38.54</td>
<td>14233</td>
</tr>
</tbody>
</table>

Table 12: 10-Year Set  
(n=328)

<table>
<thead>
<tr>
<th>Trading Cost</th>
<th>Tax-Free Trading</th>
<th>Realization Trading</th>
<th>Expected Return Trading</th>
<th>Mark-to-Market Trading</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>End Value</td>
<td>End Value</td>
<td>End Value</td>
<td>End Value</td>
</tr>
<tr>
<td>0%</td>
<td>37443.12</td>
<td>0</td>
<td>24.86</td>
<td>27401</td>
</tr>
<tr>
<td>0.25%</td>
<td>37224.75</td>
<td>169.25</td>
<td>25.96</td>
<td>27018</td>
</tr>
<tr>
<td>5%</td>
<td>36998.21</td>
<td>373.24</td>
<td>28.03</td>
<td>26079</td>
</tr>
<tr>
<td>2.5%</td>
<td>35272.01</td>
<td>1638.40</td>
<td>31.90</td>
<td>23997</td>
</tr>
<tr>
<td>5.00%</td>
<td>33245.72</td>
<td>3165.60</td>
<td>34.62</td>
<td>22292</td>
</tr>
</tbody>
</table>

Table 13: 15-Year Set  
(n=170)

<table>
<thead>
<tr>
<th>Trading Cost</th>
<th>Tax-Free Trading</th>
<th>Realization Trading</th>
<th>Expected Return Trading</th>
<th>Mark-to-Market Trading</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>End Value</td>
<td>End Value</td>
<td>End Value</td>
<td>End Value</td>
</tr>
<tr>
<td>0%</td>
<td>93139.7</td>
<td>0</td>
<td>24.28</td>
<td>55815.1</td>
</tr>
<tr>
<td>0.25%</td>
<td>92547.8</td>
<td>392.39</td>
<td>24.79</td>
<td>54933.5</td>
</tr>
<tr>
<td>5%</td>
<td>91953.4</td>
<td>781.48</td>
<td>25.18</td>
<td>54201.1</td>
</tr>
<tr>
<td>2.5%</td>
<td>87396.3</td>
<td>3780.41</td>
<td>25.68</td>
<td>50257.7</td>
</tr>
<tr>
<td>5.00%</td>
<td>82080.4</td>
<td>7265.48</td>
<td>26.94</td>
<td>46651.1</td>
</tr>
</tbody>
</table>

An increase in trading costs increases the effective tax rate under all methods. Rising trading costs have the greatest impact on the ef-

112 The extraordinarily high effective tax rates under the mark-to-market method in the 15-year set result from the assumption of internal funding and the effect internal funding has on after-tax returns on a few stocks with abnormally high rates of return. By divesting shares to pay the tax, the end value of the account is greatly reduced when the rate of return on a stock is high. If nine stocks that had a tax-free rate of return of more than 25%...
effective tax rate in the 10-year set. For example, the effective tax rate under the expected return method rises from 25.9% at a zero trading cost to 34.6% at a 5% trading cost in the 10-year set. The effective tax rate under the expected return method rises from 33.6% to 38.5% in the five-year set and from 25.3% to 30.4% in the 15-year set at zero and 5% trading costs. The sensitivity of the effective tax rate in the 10-year set to rising trading costs is consistent with the earlier finding that the timing option is of greatest relative value in the 10-year set.

The increase in the effective tax rate with rising trading costs is attributable to several sources: (1) a reduction in the average tax-free rate of return,\textsuperscript{113} (2) the deterrence of strategic trading, and (3) an increase in cost and a decrease in return of strategic trades that continue to be made. The first two sources are benign, but the third is not because additional amounts spent in executing strategic trades represent a deadweight loss to the investor and the fisc. The amount of this deadweight loss is large at high levels of trading costs under the realization and the expected return methods because strategic trading persists at high levels of trading costs for long-term investments.

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|}
\hline
Trading cost & Realization & Expected & Mark-to-market \\
\hline
0 & 27.887 & 28.68 & 37.95 \\
.25% & 28.720 & 29.56 & 39.61 \\
.50% & 29.361 & 30.26 & 39.70 \\
2.50% & 31.863 & 33.08 & 40.39 \\
5.00% & 33.943 & 35.35 & 40.91 \\
\hline
\end{tabular}
\caption{Effective tax rates under different methods and trading costs.}
\end{table}

are excluded (out of a total of 170), the effective tax rate under the mark-to-market method drops by 5% and the effective tax rate under the other methods rise.

The internal funding assumption has another interesting effect when high return stocks are included in the pool. I recorded the value of taxes paid on an investment account accruing interest at the government's short-term borrowing rates. If all stocks are included in the pool, much less tax is paid under the mark-to-market method than under the other methods because of the depletion of high-return accounts. At zero trading cost, the average taxes paid on an investment in the 15-year set was $33,888 under the realization method, $33,621 under the expected return method and $31,839 under mark-to-market. Thus, the most favorable rule to the taxpayer—realization—also collected the most taxes on average for the government. If the nine high return stocks are excluded, these positions are reversed—$19,947 is collected on average under realization, $20,098 under expected return and $24,141 under mark-to-market. One cannot conclude from this that the government does best by taxing investments least. First, these findings depend on the assumption of internal funding, for it is the sale of stock to pay the tax that depletes investments in high return stocks. Second, even if one assumes internal funding, mark-to-market produces less tax because it is assumed that tax eventually is paid on supranormal returns under the realization and expected return method. In reality, investors probably retain stock with extraordinary untaxed gains until death or dispose of the stock in ways that avoid recognition of the gain. Third, this assumes that taxes earn interest at the government's short-term borrowing rate, which obviously is less than the average rate of return on stocks.

\textsuperscript{113} This is one reason that the effective tax rates at high levels of trading cost exceed those without trading shown in Table 2.
A comparison of average trading costs with and without strategic trading reveals the magnitude of this deadweight loss. In the 10-year set for stocks held for the entire 10-year period at a 2.5% cost, average trading costs were almost twice as large with strategic trading. Under the realization method, $3,066 was spent with strategic trading compared to $1,633 spent without, or 88% greater trading costs. Under the expected return method, $3,163 was spent with strategic trading compared to $1,589 spent without, or 99% greater costs on average. At a 5% cost, trading costs were over 50% greater with strategic trading—$4,790 compared to $3,138, or 53% greater under the realization method, and $4,864, compared to $3,046 or 60% greater under the expected return method.

The percentage difference in trading costs is not as great in the 15-year set. At a 2.5% cost, trading costs were approximately 40% larger with strategic trading under the realization and expected return methods—$5,640 compared to $4,026 or 40% greater under the realization method, and $5,727 compared to $3,931 or 46% greater under the expected return method. Not only does the additional cost of strategic trading drop in percentage terms in the 15-year set—from 88% under the realization method in the 10-year set at 2.5% cost to 40% in the 15-year set—but also the additional cost of strategic trading increases only slightly in absolute dollars—from $1,433 in the 10-year set at a 2.5% cost to $1,614 in the 15-year set. This pattern is consistent with the earlier finding that the relative impact of strategic trading declines significantly in the 15-year set. Less is gained from the timing option when the investment horizon increases from 10 to 15 years because the combination of random movement of stock prices around a gradually rising mean price results in most investment ac-

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114 Column three, line one of the following table and column six, line four of Table 12.

<table>
<thead>
<tr>
<th>Realization</th>
<th>Expected Return</th>
</tr>
</thead>
<tbody>
<tr>
<td>trading cost</td>
<td>eff. tax rate</td>
</tr>
<tr>
<td>2.5%</td>
<td>33.26</td>
</tr>
<tr>
<td>5.0%</td>
<td>33.88</td>
</tr>
</tbody>
</table>

115 Column six, line one of the table in note 114 and column nine, line four of Table 12.

116 Column three, line two of the table in note 114 and column six, line five of Table 12.

117 Column six, line two of the table in note 114 and column nine, line five of Table 12.

118 Column three, line one of the following table and column six, line four of Table 13.

<table>
<thead>
<tr>
<th>Realization</th>
<th>Expected Return</th>
</tr>
</thead>
<tbody>
<tr>
<td>trading cost</td>
<td>eff. tax rate</td>
</tr>
<tr>
<td>2.5%</td>
<td>28.01</td>
</tr>
<tr>
<td>5.0%</td>
<td>28.74</td>
</tr>
</tbody>
</table>

119 Column six, line one of the table in note 118 and column nine, line four of Table 13.
counts hitting the lowest price in the 15-year investment period prior to Year 10. Few strategic trades are made after Year 10.

The cost of strategic trading strongly correlates with the volatility of the monthly return on stocks. Chart 12 shows the percentage difference between average trading costs for stocks at each level of volatility with and without strategic trading with costs at the 2.5% and 5% percent levels in the 10-year set.

![Chart 12](Image)

**Chart 12**

Additional cost of strategic trading by volatility

The findings reported in this Article raise significant doubts about the wisdom of adopting the expected return method for all but the most liquid assets. If trading costs are low and strategic trading is not otherwise constrained, a move from the realization method to the expected return method does not significantly change the average tax that an investment can be expected to bear. In other words, such a move does little to improve the neutrality of the tax on investment decisions. What may be worse, if trading costs are high but not prohibitively so, they may be significantly higher under the realization and expected return methods than under the mark-to-market and retrospective methods because of strategic trading, which can persist at fairly high levels of costs for long-lived investments. The findings also raise questions about whether the retrospective method offers sufficient improvement in tax neutrality for highly liquid assets to justify the costs of implementing such a method. This study hardly resolves these questions. A case still might be made for the expected return
method by showing that the capital loss limitation or other factors constrain strategic trading. In addition, the retrospective method still seems a promising way to tax assets that are costly to trade.

The findings raise another issue of some importance. The large potential difference between effective tax rates under the realization or the expected return method and the mark-to-market method suggests that care should be taken in where the boundaries for mark-to-market accounting is drawn. Mark-to-market accounting currently includes exchange listed options\textsuperscript{120} and futures contracts,\textsuperscript{121} but not over-the-counter derivatives. If strategic trading exists in these derivatives, then they enjoy a significant competitive tax advantage over their exchange-traded counterparts. This is true even if over-the-counter derivatives are taxed under the expected return method (as increasingly they will be). Questions deserving attention include whether strategic trading exists or is feasible in the over-the-counter markets, how far mark-to-market accounting can be extended in these markets if strategic trading is a problem, and whether retrospective taxation is a viable alternative where mark-to-market accounting cannot be extended.

\textsuperscript{120} IRC § 1256(b)(3), (4).
\textsuperscript{121} IRC § 1256(b)(1).
I. THE BINOMIAL DISTRIBUTION MODEL

This model takes all possible outcomes of an investment and discounts them by their probability to determine the expected return. To keep outcomes at a workable number, I assumed a binomial distribution of returns with a supranormal return of $x$ having a probability of $Pr(x)$ and a subnormal return of $y$ having a probability of $Pr(y)$ each period. The average or expected return is $r$.\textsuperscript{122} The return is stated as the ratio of end value to beginning value. For example, an annual rate of return of 10\% for one year produces a return of 1.1. Varying $x$ and $y$ varies the volatility and other characteristics of the investment. If an investment $i$ is held for one period, it has two possible end values—$ix$ and $iy$—with probabilities of $Pr(x)$ and $Pr(y)$. If an investment $i$ is held for two periods, it has four paths to three possible end values $ix^2$, $ixy$ and $iy^2$, with probabilities of $Pr(x)^2$, $2*Pr(x)Pr(y)$ and $Pr(y)^2$. It is helpful to think of the paths to the end values as a tree, which is as follows for a four-period investment:

Over $n$ periods, the number of possible end values equals $n+1$ with $2^n$ paths to those ends. Thus, a six-period investment has 64 paths to seven end values that range from $ix^6$ to $iy^6$ with the greatest number of paths (20) leading to $ix^3y^3$ with a probability of $20Pr(x)^3Pr(y)^3$. The number of paths to each end point equals the sum of the paths to the

\begin{align*}
\text{Pr}(x) &= r - ylx + y \text{ and } Pr(y) = 1 - Pr(x).
\end{align*}
preceding points (or single point at the outer edges) in the previous period.

The expected return on an investment is the sum of the possible returns in the last period discounted by their probability. Once tax is taken into account, the expected return varies depending on an investor's strategy regarding sales prior to term. Often a strategy of selling after a low return (y) or loss and reinvesting the proceeds improves the expected return. To determine the effect of sales, I calculated the expected return given a strategy of selling at certain points and then reinvesting the proceeds for the remainder of the original term in a similar investment.\textsuperscript{123} I tested several plausible strategies. For example, for a three-period investment, I determined the expected return if the investor planned to sell after a low return in Period 1 or Period 2, if he planned to sell after a low return in Period 1 but not Period 2, or if he planned to sell after two periods of low return. I assumed that the expected return on the investment equalled the return under the best strategy.

The number of possible strategies grows exponentially with the number of periods. For two periods there are four strategies—sell after a high return in one, sell after a low return, always sell or always hold. For three periods there are 16, for four periods 64 and so on.\textsuperscript{124} Fortunately, only some of these strategies plausibly maximized return and so merited evaluation. For instance, it never would be profitable to sell at a gain. Nevertheless, for most of the analysis, I limited the investment to seven periods because it was too cumbersome to account for the rapidly increasing number of plausible sale strategies as the number of periods increased.

I assumed expected returns were constant over time as investment value changed. Thus, the rate of imputation under the expected return method and the rate of interest charged under the retrospective method were constant. I ignored inflation. Most accept that inflationary returns do not represent income because they do not improve purchasing power. Inflation might be handled through upward basis adjustments, downward adjustments in the imputation rate under the expected return method and upward adjustments in the lending rate.

\textsuperscript{123} It is not appropriate to assume reinvestment at a risk-free rate for the balance of the original term of an investment when that term is more than one period, for this artificially discourages sales because the investor loses whatever tax preference attaches to the original investment. Thus, an investor might not sell stock that dropped in value if his only choice was to reinvest in Treasury bonds paying current interest since he would lose the advantage of deferral of tax on possible future gains. If reinvestment is for one period, I assume it is at the risk-free rate of return subject to current tax. This produces the same result as applying each of the methods examined here to a one-period investment.

\textsuperscript{124} If \( p \) is the number of periods, the number of strategies is \( 2^p(p-1) \).
under the retrospective method. I do not make such adjustments because my goal was to compare alternative tax rules. Adjusting for inflation would not affect the comparison.

I assumed that the imputation rate under the expected return method and the rate of interest charged under the retrospective method equaled the normal after-tax rate of return (or \( r(1 - t) \), where \( r \) is the normal rate and \( t \) is the tax rate). The two rates would be equal for an investor whose pretax cost of borrowing and return on investing were equal. For many taxpayers, the cost of borrowing exceeds available rates of return, though available rates of return define the opportunity cost of borrowers. I do not account for such differences. The practical reason for this assumption is that it made the simulation easier because dollars could be treated as of equal value whether they are paid or owed. There is some theoretical justification for the assumption. The rules examined here most likely would apply to new financial products, which are primarily held by banks and large corporations with near-equal expected costs and returns on capital. Further, any credit risk would be borne by the federal government in the form of deferred taxes. The government may bear less risk than ordinary lenders because of its unique power in collecting taxes. The credit risk when the collection of taxes is deferred to the liquidation of an investment is somewhat mitigated by the fact that the sale provides the means to pay the tax. However, some unaccounted for difference in the two rates remains. Generally, this unaccounted for difference lessens the relative attractiveness of the retrospective method and the realization rule compared to the mark-to-market method or the expected return method because expected after-tax returns under the first two methods would be slightly higher than stated here because taxpayers would factor in the possibility of consuming returns and then defaulting on the tax.

II. Historic Stock Data

I also tested the effects of expected return and retrospective taxation on actual returns using historic stock data. I started with a sample of 500 stocks with returns from January 1, 1960 to December 31, 1992 from the CRSP data base.\(^\text{125}\) Stocks were drawn if they appeared at any time during the 33 year period. I drew monthly price data and all distribution data for each stock. The findings reported in this Article are based only on the data from 1960 to 1990.

The simulation tracked returns of an investment of $10,000 in each stock that appeared at the beginning of a test period. The investment

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\(^{125}\) See note 72.
was held through the period or until the stock ceased to trade. There were six consecutive five-year periods starting on January 1, 1960. There were three consecutive 10-year periods starting on January 1, 1960. There were two consecutive 15-year periods starting on January 1, 1960. I used consecutive rather than overlapping periods (which would have greatly increased the number of periods that may have been observed) to avoid over-weighting the data from the periods in the middle of the 33-year sample. I compared tax-free returns with after-tax returns under the realization rule, the expected return method, the retrospective method and a mark-to-market rule that required gains and losses to be realized at the end of each tax period.

I tried to keep the simulation as realistic as possible. Investments were made in full shares and I carried a cash account for excess funds with interest credited at the current rate for low-risk corporate bonds. In imputing income under the expected return method and imputing deferred taxes under the retrospective method, I credited dividends with taxes paid.

Nondividend distributions were taxed based on their stated character. Generally, I was able to respect existing rules on character. However, distributions of stock other than the underlying security were treated as taxable cash distributions even if the distribution was identified as tax-free. Furthermore, under the retrospective method, it was necessary to treat some distributions that were denominated as taxable only if in excess of basis as distributions in partial liquidation instead (with basis prorated) because of anomalies caused by a zero or near-zero basis or investment in calculating the internal rate of return on an investment.

Under the expected return and mark-to-market methods, taxes must be paid on unrealized gains. Initially, I simulated both internal and external funding of these taxes. External funding, in effect, meant that taxes were paid by borrowing, reflected in a negative cash account on which interest was charged. Internal funding meant that shares were sold to pay the tax if the cash account was not sufficient. Eventually, I settled on a consistent assumption of internal funding because external funding created significant distortions in outcome under the mark-to-market method, showing taxes at confiscatory rates (and in a few cases producing negative account balances), when stocks that rose in value for an extended period of time subsequently crashed.

As dividends and interest accrue on the tax account, sufficient funds may be collected to purchase new shares of stock. At first, I reinvested funds only when the original stock in the account was sold, but I found that this clouded the comparison of tax-free investing and
mark-to-market accounting with the other three rules because the lack of strategic sales under the first two rules meant significant cash accounts were carried risk-free. I modified the programs to reinvest funds whenever they were sufficient to purchase a new share, but this raised problems of handling multiple accounts of shares. Maintaining unlimited multiple accounts greatly increases computational time and is unrealistic. The simplest solution (but not what the law does) is to blend the new and old shares into a single account with an averaged basis. Blending new and old shares has little effect under the mark-to-market method because marking to market equalizes year-end basis. Blending new and old shares works tolerably well under the realization and expected return methods, although averaging basis affects opportunities for strategic trading under both methods and affects the imputation rate under the expected return method. New and old shares cannot be blended under the retrospective method because the holding period is crucial to the assessment of tax. To maintain comparability among the methods, I settled on a program for all but the mark-to-market method that maintained shares in up to 10 accounts and sold the stock in the smallest account when necessary to make space for new stock. I fudged on sales in cleaning out an account by not charging costs on the sale.

For the rate of imputation under the expected return method and the lending rate under the retrospective method, I used the interest rate on long-term federal bonds published by Treasury as the long-term AFR under § 1274(d). For years from 1970 to 1984, I used the annual rate published by the Federal Reserve Board for 10-year Treasury Bonds. Interest was credited on the cash account and interest was compounded on transaction costs at the rate of three-month Treasury Bills.

The calculation of the effective tax rate is not obvious. Normally the effective tax rate on an investment is computed by dividing the after-tax rate of return by the pretax rate of return. This is simple to do when analyzing a single investment with a single payoff at the end from which tax is paid—simply compare the rates of return using the end value before and after the tax is subtracted. It is more complex when analyzing a pool of investments with varying lives and tax paid at various times over the life of the investment.

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It is impractical to compute the effective tax rate for individual stocks by adding positive tax payments to the end value of an account (or subtracting tax refunds) to determine what the return rate would have been without taxes. If a stock drops significantly in value, subtracting tax refunds will produce a negative end value, making it impossible to calculate the rate of return. The following table illustrates this point with a single stock from 1970.127

<table>
<thead>
<tr>
<th>period</th>
<th>tax-free end value</th>
<th>realization rate of return</th>
<th>tax-free realization</th>
<th>realization rate of return</th>
<th>realization tax refunds plus interest</th>
<th>realization end value subtracting tax refunds</th>
</tr>
</thead>
<tbody>
<tr>
<td>1970</td>
<td>$577.00</td>
<td>-43.44%</td>
<td>$1966.90</td>
<td>-27.74%</td>
<td>$-6843.65</td>
<td>$-4876.75</td>
</tr>
</tbody>
</table>

The stock dropped in value from 1970 to 1975—an investment of $10,000 in 1970 was worth only $577 at the end of 1974. The investor was better off because of the tax deduction—the end value under a rule of realization increases to $1,966.90. However, the cost to the government from the tax exceeds the benefit to the investor—taxes refunded to the investor including interest equalled $6,843.65 at the end of the period. Refunds were squandered by the investor because they were reinvested in the stock, which continued to decline in value. Subtracting the tax refunds and interest from the end value of the account produces a negative end value (−$4,876.75).

The effective tax rate might be calculated for individual stocks by comparing the rate of return on an account maintained tax-free with the rate of return on an account that was taxed. In the example above, this produces an effective tax rate of 36.15%.128 Negative taxes when the rate of return is negative produce a positive effective tax rate. But this approach to calculating the effective tax rate does not work well when the rate of return is near zero. At near-zero return rates, differences in the end value of an account can produce extraordinarily high and low effective tax rates. The following table shows the end value, rate of return, and effective tax rate in a tax-free world and under each of the tax rules for an investment in another security from 1975 to 1980.129

<table>
<thead>
<tr>
<th>Tax-Free</th>
<th>Realization</th>
<th>Expected Return</th>
<th>Mark-to-Market</th>
<th>Retrospective</th>
</tr>
</thead>
<tbody>
<tr>
<td>end value</td>
<td>$9447.94</td>
<td>$9926.47</td>
<td>$9938.97</td>
<td>$10511.47</td>
</tr>
<tr>
<td>rate of return</td>
<td>−1.1281</td>
<td>−0.14734</td>
<td>−0.12222</td>
<td>1.001532</td>
</tr>
<tr>
<td>effective tax rate</td>
<td>86.94</td>
<td>89.17</td>
<td>188.78</td>
<td>77.51</td>
</tr>
</tbody>
</table>

127 The table illustrates the problem with respect to a security of Computer Sciences Corporation in the CRSP data base with identification number 20536310.
128 1−(−.277/−.434).
129 The share is identified by Cusip 16278910.
A $1,063 improvement in end value under the mark-to-market method produces a phenomenal effective tax rate of 188.7%. A similar dollar difference in end value at a rate of return further from zero would alter the effective tax rate much less. To calculate the overall effective tax rate by averaging the effective tax rate on individual stocks gives the few stocks with near-zero returns too much weight. This is problematic because returns on individual stocks may differ for reasons that essentially are random. In the example, the return under the mark-to-market method is highest because the stock rises in value and then drops. Why an investor may profit under the mark-to-market method when this occurs is explained above.\textsuperscript{130}

To overcome this problem, it is necessary to calculate the effective tax rate either by calculating the effective tax rate on the average rate of return for stocks in the set or by calculating the effective tax rate on the rate of return on the average end value of stocks in the set. Each method has different drawbacks. The second method requires that stocks be compared at the same point in time since values cannot be compared across time without a discount rate. Thus, stocks that cease trading prior to the end of the period must be discarded. The first method results in an effective tax rate below the statutory rate even though the tax is neutral. Consider a simple example using the binomial distribution model: a two period investment where $y = 1.4$, $y = -0.7$, and the tax rate is 40%. If one determines the effective tax rate using the rate of return on the average end value of the four paths in a world with and without tax, then the effective tax rate under the expected return method is 40% without strategic trading.\textsuperscript{131} If instead one determines the effective tax rate by averaging the rates of return on the four paths in a world with and without tax, then the effective tax rate under the same method drops to 27.42%.\textsuperscript{132}

\textsuperscript{130} See note 102.

\textsuperscript{131} The average end value without tax is $121 and $112.36 with tax, which produces rates of return of tax-free 10% and 6% with tax.

\textsuperscript{132} The average tax-free rate of return is 6.24%; the average rate of return with tax is 4.52%.