**PRICE CAPS IN MULTI-PRICE MARKETS**

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**Abstract**

Many consumer markets feature a multi-dimensional price. A policymaker – a legislator, a regulator or a court – concerned about the level of one price dimension may decide to cap this price. How will such a price cap affect other price dimensions? Will the overall effect be good or bad for consumers? For social welfare? Price caps can be beneficial when sellers set prices in response to consumer misperception. The scope for welfare-enhancing regulation depends on the type (and direction) of the underlying misperception, as well as on market structure.

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1. Introduction

A. Motivation

The Credit Card Accountability, Responsibility and Disclosure Act of 2009 (the CARD Act), and its implementing regulations, imposed restrictions on certain dimensions of the credit card price. In particular, late fees were subjected to a de facto price cap. Other fees and interest rates were also curtailed. A few years later, the Dodd-Frank Act restricted the permissible magnitude of prepayment penalties in mortgage contracts. Other examples of price caps are easy to find. Usury laws cap interest rates. Courts applying the Penalty Doctrine imposed de facto caps on cellphone early termination fees. The European Union caps roaming fees and international calling rates. The Singapore Telecommunication Act of 2000 caps the price that hotels can charge for international phone calls. Etc.

In these examples, lawmakers, responding to concern about an excessively high price, resolved to cap the suspect price. The lawmakers did not fully account, however, for the possibility of unintended consequences. In particular, credit cards, mortgages, cellular service and hospitality services are all multi-dimensional products with multi-dimensional prices. When the law caps one price dimension, we cannot assume that other price dimensions will remain unchanged. If sellers react to the new law by increasing other prices, then it is no longer clear that the law will achieve its stated purpose.

Will the price cap increase social welfare? Will it make consumers better off? To answer these questions we need to first understand the forces driving the pre-cap pricing structure. If prices were efficient, designed to provide optimal incentives, then a price cap will likely reduce social welfare and hurt consumers. These distortions might be
exacerbated in a multi-price market, where a price cap on one dimension can lead to adjustment away from the efficient level also on other price dimensions. If, on the other hand, pre-cap prices were designed not to maximize efficiency but to exploit consumer biases, then legal intervention may increase welfare and help consumers.

B. Framework

I study a model with two product, and price, dimensions: The first dimension, labeled Dimension X, represents a binary decision – to purchase, or not to purchase the product. Think of a decision whether or not to book a room for one week in a certain hotel (the duration of your stay is determined exogenously). The consumer enjoys a base utility from the hotel room and pays a base price, $p_x$, for the hotel room. The second dimension, Dimension Y, represents a continuous use decision. Think of a decision to order room service – the consumer could order room service for any number of meals during her stay at the hotel. The consumer enjoys a per-use, or per-meal, utility from in-room dining and pays a per-use, or per-meal, price, $p_y$.

Misperception afflicts only the use dimension (Dimension Y). I consider two types of consumer misperception: utility misperception and price misperception. With utility misperception, the consumer under- or overestimates the utility from Dimension Y. For example, the consumer might underestimate the utility from in-room dining. With price misperception, the consumer under- or overestimates the per-use price, $p_y$. For example, when booking the hotel room, the consumer might underestimate the room-service prices in the hotel.
C. Second-Best Optimal Prices

When demand is biased by consumer misperception, pre-cap prices will be inefficient. To assess this inefficiency, it is helpful to first characterize the efficient prices in a multi-dimensional pricing scheme. I refer to these prices as second-best optimal, i.e., optimal given the misperception. (This paper considers different types, directions and levels of misperception, but the misperception is exogenous to the analysis. For a discussion of endogenous misperceptions – see, e.g., Bar-Gill, 2012.) There are two plausible meanings of second-best optimal prices in this context: (i) prices that maximize the consumer surplus, and (ii) prices that maximize social welfare. The two meanings bear different normative implications and different policy prescriptions in certain cases. I first describe second-best prices that maximize the consumer surplus. I then explain how the results change when we move to social welfare maximization.

In the absence of misperception, cost-based pricing is efficient: each price dimension should be set equal to the cost of providing the corresponding product or service dimension. When misperception is introduced, cost-based pricing is no longer optimal.

Interestingly, while the second-best optimal prices depend on the type of misperception – utility misperception vs. price misperception – they do not depend on the direction of the misperception. Optimal prices are the same for both under- and overestimation. And this result holds for both utility misperception and price misperception.

Utility misperception. With utility misperception, it is second-best optimal to increase the price associated with the misperceived dimension, \( p_y \), above cost and reduce the price associated with the accurately perceived dimension, \( p_x \), below cost. This pricing
pattern holds for both under- and overestimation of utility, but for different reasons. When consumers underestimate the utility from in-room dining, demand for hospitality services will be too low (since in-room dining is one dimension in the bundle of services provided by the hotel). Demand can be efficiently increased by shifting pricing towards the underestimated dimension, i.e., by increasing room-service prices and reducing the base-rate charged for the hotel room itself. A consumer who underestimates the utility from in room-dining will also underestimate the number of in-room meals that she will order. Accordingly, the consumer will underestimate the effect of higher room-service prices. While the reduction in \( p_x \) will be fully appreciated, the effect of the corresponding increase in \( p_y \) will be underestimated. Therefore, the perceived total price will go down, and demand will go up.

When consumers overestimate the utility from in-room dining, demand for hospitality services will be too high. But, again, increasing room-service prices and reducing the base-rate improve things – this time by reducing demand. A consumer who overestimates the utility from in room-dining will also overestimate the number of in-room meals that she will order. Accordingly, the consumer will overestimate the effect of higher room-service prices. The consumer will accurately perceive the reduction in \( p_x \), and overestimate the effect of the corresponding increase in \( p_y \). Therefore, the perceived total price will go up, and demand will go down.

*Price misperception.* With price misperception, second-best prices will, again, be independent of the direction of the misperception. But these second-best prices will be very different from those obtained for utility misperception: It is optimal to reduce the misperceived price, \( p_y \), below cost and to increase the accurately perceived price, \( p_x \),
above cost. With both under- and overestimation, it is optimal to shift pricing away from the misperceived price dimension and towards the accurately perceived price dimension. Such a shift reduces the difference between the actual total price that the consumer will pay for the product and the price that the consumer thinks she will pay for the product.

_Social welfare maximization_. When we are maximizing social welfare, rather than consumer surplus, second-best prices are different, but only with utility overestimation and price underestimation. These misperceptions result in excessive demand. The optimal response is to increase the base price, while keeping the base price at cost – to avoid use-level distortions. This response achieves the first-best level of welfare. (Note that sellers are making strictly positive profits, which means that consumer surplus is not maximized.)

D. Pre-cap Equilibrium Prices

To assess the welfare cost of misperception, and the scope for welfare-enhancing price caps, I now compare the second-best prices to the pre-cap, equilibrium prices. (I use “welfare,” in “welfare cost” and “welfare enhancing,” loosely – to refer to both social welfare and consumer surplus.) I begin with perfect competition and then consider the implications of monopoly and market power.

We saw that the second-best prices depend on the type of misperception, but not on the direction of the misperception, at least not when we are maximizing consumer surplus. In contrast, the pre-cap, equilibrium prices are very much affected by the direction of the misperception, but not so much by the type of misperception. For both utility underestimation and price underestimation, the equilibrium pre-cap, per-use price,
$p_y$, exceeds the per-use cost; and the equilibrium base-price, $p_x$, is below the base cost. In the hotel example, in-room dining prices are set above the cost to the hotel of providing this service, while the basic room rates are set below cost. Conversely, for both utility overestimation and price overestimation, the pre-cap, per-use price, $p_y$, is below the per-use cost; and the base-price, $p_x$, exceeds the base cost.

**Underestimation.** When the utility from in-room dining is underestimated, we saw that the second-best room-service prices are above cost. The pre-cap equilibrium room-service prices will be even higher. By reducing the basic room rates and increasing room-service prices, the hotel increases the perceived (net) value of its product and counteracts the utility underestimation. But, at the same time, the hotel reduces the actual value by distorting incentives to utilize its in-room dining services – high room-service prices imply fewer room-service orders. Second-best optimal pricing balances these two effects. The hotel, however, cares only about perceived value and thus sets room-service prices inefficiently high.

When room-service prices themselves are underestimated, the pre-cap equilibrium room-service prices will again be above cost. As with utility underestimation, the hotel increases the perceived (net) value of its product by shifting prices towards the underestimated dimension. Note, however, that the difference between the equilibrium price and the second-best price, and correspondingly the space for a welfare-enhancing price cap, is significantly larger with price underestimation. While the equilibrium price is above-cost for both types of misperception, the second-best price is significantly lower with price misperception.
*Overestimation.* When the utility from in-room dining is overestimated, the consumer overestimates the number of room-service orders and thus overestimates the importance of room-service prices. To minimize the perceived total price of its product, the hotel responds to the misperception by reducing room-service prices below cost, and increasing the basic room rate above cost. Similarly, when room-service prices themselves are overestimated, the hotel shifts prices away from the overestimated, room-service dimension and to the accurately perceived basic room rate.

When utility from in-room dining or the price of in-room dining are overestimated, room-service prices are too low and there is no point in capping them. But, since pricing will shift to the accurately perceived basic room rate, this price will be too high and welfare can potentially be enhanced by capping it. While the pre-cap equilibrium room rate is similarly above cost for both utility and price overestimation, second-best prices are very different. Specifically, the second-best room rate is significantly lower with utility overestimation and, accordingly, there is more room for a welfare-enhancing price cap when the object of misperception is utility rather than price.

*Summary.* The nature and scope of welfare-enhancing regulation depends on both the type and direction of misperception. With underestimation – of both utility and price – it is the misperceived price (or the price associated with the misperceived dimension), $p_y$, that needs to be capped. With overestimation it is the accurately perceived price (or the price associated with the accurately perceived dimension), $p_x$, that needs to be capped.

The question of scope is of particular importance when imperfectly informed lawmakers might set the price cap too low and reduce welfare. When the problem is
underestimation, price misperception gives the imperfectly informed lawmaker a larger target to aim at. In particular, with price underestimation, the lawmaker can set the cap at cost (since the pre-cap equilibrium price is above cost and the second-best price is below, or at, cost), when information about cost is more readily available. With utility underestimation, a price cap equal to cost is too low (since both the pre-cap equilibrium price and the second-best price are above cost). Conversely, when the problem is overestimation, utility misperception gives the lawmaker a bigger target, if we are maximizing consumer surplus. In particular, with utility overestimation the lawmaker can set the cap at cost, whereas such a cap might reduce consumer surplus with price overestimation. If we are maximizing social welfare, a price cap equal to cost is too low for both utility and price overestimation.

E. Market Power

I have thus far assumed that sellers operate in a perfectly competitive market. I now replace competition with monopoly and examine how market power alters the positive and normative implications of price regulation. The model developed in this paper assumes that the X and Y dimensions are separable. Therefore, the price associated with the misperceived dimension, $p_y$, does not depend on market structure, and many of the results derived in the perfect competition case apply in the monopoly case as well. The main difference is that the monopolist will set a higher price on the accurately perceived dimension. The higher $p_x$ affects welfare in a subtle way: With utility underestimation and price overestimation, monopoly pricing reduces welfare – the misperception results in inadequately low demand and the high $p_x$ reduces demand even further. With utility
overestimation and price underestimation, demand is excessively high in a competitive market. The high $p_x$ set by the monopolist counteracts the inflated demand. But, while the high $p_x$ avoids purchases that generate a social loss, it might also deter purchases that generate a social gain. Accordingly, with utility overestimation and price underestimation the net welfare effect of monopoly pricing is indeterminate.

Now turn to the price-cap itself. At the descriptive level, market power moderates the effect of a price cap on the unregulated price. Consider underestimation – of either utility or price – which could justify a cap on $p_y$. In a competitive market, a cap that reduces $p_y$ forces the seller to increase $p_x$, so that the seller covers her overall costs. This unintended consequence – an increase in the unregulated price – is less likely to occur in a monopolistic market. In the pre-cap world, the monopolist may have decided to increase the underestimated price, since the increase did not significantly reduce demand for the monopolist’s product. Increasing an accurately perceived price, in response to a cap on the underestimated price, would cost more in terms of reduced demand. Therefore, the monopolist may decide not to increase the unregulated price, or to increase it by a smaller amount. At the normative level, if the regulated price goes down and the unregulated price does not go up (or not by much), demand will increase. When demand was too low pre-cap, as with utility underestimation, this increased demand is socially desirable. However, when the first-order effect of the misperception is to artificially inflate demand, as with price underestimation, high (pre-cap) monopoly pricing may efficiently offset this effect. A price cap that increases demand will then reduce welfare.
F. Related Literature


These papers by and large do not consider price caps. The important exceptions are DellaVigna and Malmendier (2004), Heidhues and Koszegi (2010) and Armstrong and Vickers (2012). DellaVigna and Malmendier briefly discuss the potential welfare benefits of price regulation. They study naiveté about time preferences, which is related to the utility misperception studied here. This paper extends the analysis in DellaVigna and Malmendier (2004) by considering different types (and directions) of misperception and by comparing the positive and normative implications of the different types (and directions) of misperception. Heidhues and Koszegi (2010) focus on credit contracts, but their model could be generalized. They demonstrate the potential welfare benefits of price regulation. Like DellaVigna and Malmendier (2004), Heidhues and Koszegi (2010) study naiveté about time preferences. Heidhues and Koszegi study only the perfect competition case and thus do not identify the effects of market structure on the welfare implications of the price cap.

The model in Armstrong and Vickers (2012) appears to cover both utility and price misperception, but in a way that masks the positive and normative differences between

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1 Gabaix and Laibson (2006) mention price caps in one short paragraph, with no analysis.
the two types of misperception. Also Armstrong and Vickers study only the perfect competition case. On the other hand, the model in Armstrong and Vickers (2012) is more general than the model in the current paper, since they allow for heterogeneity in consumer misperception (studying markets where some consumers suffer from misperception, but others do not). Armstrong and Vickers show that a price cap can increase welfare, in large part by limiting the cross-subsidization of sophisticated consumers by less sophisticated consumers.

This paper builds on and extends the analysis in Bar-Gill and Bubb (2012). Bar-Gill and Bubb study the effects of a price cap in a simple, linear-demand model where each price, in a two-dimensional pricing scheme, is incurred exactly once. The implication is that prices do not have incentive effects, beyond the purchase decision. The current paper relaxes some of the simplifying assumptions in Bar-Gill and Bubb (2012), generalizing and refining the results of that paper and deriving additional results. Also, while Bar-Gill and Bubb (2012) focus on price underestimation, this paper compares the positive and normative implications of different types (and directions) of misperception. Agarwal et al (2013), in addition to its very sophisticated empirical analysis of the CARD Act, includes a short theory section that uses a model similar to the one developed in Bar-Gill and Bubb (2012). Like Bar-Gill and Bubb (2012), Agarwal et al (2013) focus on price underestimation. Their treatment of market structure, or market power, is more general than Bar-Gill and Bubb (2012).
G. Roadmap

The framework of analysis is developed in Section 2. The main results are derived in Sections 3-5. Section 6 offers concluding remarks, discussing several extensions: markets where multiple product and price dimensions are subject to misperception; markets where consumers suffer from multiple types of misperception (on a single dimension); indirect forms of price regulation, beyond price-caps; and quality floors in markets where product quality (rather than price) is multi-dimensional. Proofs are relegated to an appendix.

2. Framework of Analysis

A. Basic Setup

Assume a two-dimensional product \((X,Y)\). The consumer chooses how much to consume on each dimension, i.e., the consumer chooses consumption levels \((x,y)\). For simplicity, assume that \(X\) is a binary dimension, i.e., \(x \in \{0,1\}\), with \(x = 1\) representing a decision to purchase the product and \(x = 0\) representing a decision not to purchase the product. If the consumer decided to purchase the product, she must then decide how intensely to use the product on the \(Y\) dimension, where \(y \in \mathbb{R}^+\). (The model can be extended to accommodate continuous decisions on both dimensions, e.g., when a consumer decides how much to borrow, on a credit card, in an introductory period \((x)\) and how much to borrow in the post-introductory period \((y)\).)

The assumption is that \(X\) and \(Y\) are two dimensions of a single product. Or, equivalently, that \(X\) and \(Y\) are effectively bundled, such that a consumer who purchases \(X\) from one seller will not purchase \(Y\) from another seller. Moreover, it is assumed that all sellers are offering both \(X\) and \(Y\), and that no seller can offer just \(X\) (or just \(Y\)). The idea
is that X and Y are very difficult to separate or, alternatively, that there are substantial efficiencies from bundling them together (or that bundling is very profitable for behavioral reasons).

The seller’s cost of providing the product is separable, with an independent per-unit cost for each dimension of the product. There is a fixed cost, $c_x$, of serving any consumer who chooses to purchase the product, and a per-unit cost, $c_y$, for each unit of use on dimension Y. The seller’s total cost, for a consumer who decided to purchase the product, is: $C(c_x, c_y) = c_x + yc_y$.

The (gross) value of the product to the consumer is: $v + u(y)$, where $v$ is a base-value that is distributed among consumers according to the CDF $F(v)$, and $u(y)$ is a use value that varies with use levels on the Y dimension but in a manner common to all consumers. I assume that $u'(y) > 0$ and $u''(y) < 0$.

B. The Seller’s Decisions

The seller sets a two-dimensional price, which is comprised of a per-unit price for each dimension of the product. The per-unit prices are: $p_x$ and $p_y$. The price $p_x$ will be referred to as the base price; the price $p_y$ will be referred to as the per-use price. The total price is: $P(p_x, p_y) = p_x + yp_y$. The seller’s profit per-product purchased is:

$$\pi(p_x, p_y) = P(p_x, p_y) - C(c_x, c_y) = (p_x - c_x) + y(p_y)(p_y - c_y)$$

Note that $\pi(p_x, p_y)$ is increasing in $p_x$. And we assume that $\pi(p_x, p_y)$ is also increasing in $p_y$, namely that: $\frac{\partial \pi(p_x, p_y)}{\partial p_y} = y(p_y) + \frac{dy(p_y)}{dp_y}(p_y - c_y) > 0$, which follows immediately from:
Assumption 1: Profits on the use dimension \( y(p_y) (p_y - c_y) \) are monotonically increasing in the per-use price, \( p_y \), in the relevant range.

The seller’s total profit function is:

\[
\Pi(p_x, p_y) = \pi(p_x, p_y) \cdot D(p_x, p_y)
\]

where \( D(p_x, p_y) \) represents the demand for the seller’s product, i.e., the number of consumers who purchase the product. The demand function is derived below.

The prices that the seller sets, and the profit that the seller makes, depend, among other things, on market structure. I will consider two different assumptions about the structure of the market: perfect competition and monopoly. In a perfectly competitive market, prices will be set to maximize the (net) value of the product, as perceived by consumers, subject to a zero-profit constraint: \( \Pi(p_x, p_y) = 0 \). In a monopolistic market, prices will be set to maximize \( \Pi(p_x, p_y) \).

C. The Consumer’s Decisions

The consumer makes two decisions: (1) whether to purchase the product, and (2) how intensely to use a product that is purchased. I begin by describing the use decision. The prior purchase decision is (potentially) influenced by consumer misperception. I, therefore, present the different types of misperception, before turning to the purchase decision itself.
1. Use Decision

A consumer who has decided to purchase the product will choose a use level, \( y \), that solves: \( \max_y (u(y) - yp_y) \). The First-Order Condition (FOC) is: \( u'(y) = p_y \), which implicitly defines the optimal use level as a function of the per-unit price, \( p_y \): \( y = y(p_y) \).

Let \( \eta_{y,p_y} = \frac{dy(p_y)/y(p_y)}{dp_y/yp_y} \) denote the elasticity of use levels with respect to the per-use price.

2. Consumer Misperception

I consider two different types of misperception: utility misperception, where the consumer believes that her use value will be \( \hat{u}(y) = \delta u(y) \), where \( \delta \in [0, \infty) \); and price misperception, where the consumer believes that the per-use price will be \( \hat{p}_y = \delta p_y \), where \( \delta \in [0, \infty) \). I study the two types of misperception separately. (The case where consumers might suffer from both types of misperception simultaneously is discussed briefly in Section 6.B.). Therefore, I can use the same parameter, \( \delta \), for both types of misperception. The benchmark case, where the consumer does not suffer from any misperception, is captured by \( \delta = 1 \). Underestimation is captured by \( \delta < 1 \), and overestimation is captured by \( \delta > 1 \).

Both types of misperception apply only ex ante. Ex post, when the actual use decision is made, the consumer learns her true use value, \( u(y) \), and the actual per-use price, \( p_y \), and sets the use level, \( y \), accordingly (as described in subsection 1 above). (Compare: naïve hyperbolic discounters in DellaVigna and Malmendier (2004) and Heidhues and Koszegi (2010).) But, ex ante, when making the purchase decision, the consumer thinks that she will choose a different use level:
(1) With utility misperception, the consumer thinks that she will choose a use level, \( y \), that solves \( \max_y (\delta u(y) - yp_y) \). The First-Order Condition (FOC), \( \delta u'(y) = p_y \), implicitly defines the anticipated use level as a function of the per-unit price, \( p_y \), and the misperception parameter, \( \delta \): \( y = \bar{y}(p_y; \delta) \).

(2) With price misperception, the consumer thinks that she will choose a use level, \( y \), that solves \( \max_y (u(y) - yp_y) \). The FOC, \( u'(y) = \delta p_y \), implicitly defines the anticipated use level as a function of the per-unit price, \( p_y \), and the misperception parameter, \( \delta \): \( y = \bar{y}(p_y; \delta) \).

Note that utility underestimation leads to underestimation of use levels, while price underestimation leads to overestimation of use levels. Conversely, utility overestimation leads to overestimation of use levels, while price overestimation leads to underestimation of use levels.

3. Purchase Decision

The decision whether to purchase the product depends on the (net) value of the product, as perceived by the consumer. The (net) value of the product to a consumer is:

\[
V(v, p_x, p_y) = v + u(y) - (p_x + yp_y).
\]

This (net) value might be misperceived by the consumer. Specifically, the use dimension – the per-use price, the use level and the use value – are subject to (possible) misperception. The perceived (net) value of the product is: \( \bar{V}(v, p_x, p_y; \delta) = v + \bar{u}(\bar{y}) - (p_x + \bar{p}y) \). This formulation captures the two types of misperception defined in subsection 2 above: With utility misperception, we have \( \bar{u}(\bar{y}) = \delta u(\bar{y}), \bar{p}y = p_y \) and \( \bar{y} = \bar{y}(p_y; \delta) \); with price misperception, we have \( \bar{u}(\bar{y}) = u(\bar{y}), \bar{p}y = \delta p_y \) and \( \bar{y} = \bar{y}(p_y; \delta) \).
The consumer will purchase the product iff the perceived (net) value is positive, i.e., iff \( \hat{V}(v, p_x, p_y; \delta) > 0 \). There exists a threshold value, \( \hat{v}(p_x, p_y; \delta) = (p_x + \gamma p_y) - \hat{u}(\hat{y}) \), such that consumers with \( v > \hat{v}(p_x, p_y; \delta) \) will purchase the product. Assuming a unit mass of consumers, the demand for the product is:

\[
D(p_x, p_y; \delta) = 1 - F\left(\hat{v}(p_x, p_y; \delta)\right).
\]

The perceived overall consumer surplus is:

\[
\hat{S}(p_x, p_y; \delta) = \int_{\hat{v}(p_x, p_y; \delta)}^{\infty} \hat{V}(v, p_x, p_y; \delta) f(v) dv
\]

whereas the actual overall consumer surplus is:

\[
S(p_x, p_y; \delta) = \int_{\hat{v}(p_x, p_y; \delta)}^{\infty} V(v, p_x, p_y) f(v) dv
\]

Note that prices that maximize consumer surplus must satisfy the zero-profit condition, \( \Pi(p_x, p_y) = 0 \), which implies: \( p_x + y p_y = c_x + y c_y \). (Otherwise, we could “costlessly” increase the consumer surplus by raising \( p_x \).) With this condition, the (net) value becomes: \( V(v, p_x, p_y) = v + u(y) - (c_x + y c_y) \), and the overall consumer surplus can be written as:

\[
S(p_x, p_y; \delta) = \int_{\hat{v}(p_x, p_y; \delta)}^{\infty} \left[ v + u(y(p_y)) - (c_x + y(p_y) c_y) \right] f(v) dv
\]
D. Social Welfare

1. The Social Welfare Function

Total social welfare is the sum of utilities enjoyed by consumers who choose to make a purchase, i.e., consumers with \( v > v(p_x, p_y; \delta) \), minus the cost – to the seller – of serving these consumers. The social welfare function is:

\[
W(p_x, p_y; \delta) = \int_{\phi(p_x, p_y; \delta)}^{\infty} \left[ v + u(y(p_y)) - (c_x + y(p_y)c_y) \right] f(v) dv
\]

Note that \( W(p_x, p_y; \delta) = S(p_x, p_y; \delta) \). Still, prices that maximize social welfare will, in certain cases, deviate from prices that maximize consumer surplus, since social welfare maximization requires \( \Pi(p_x, p_y) \geq 0 \) whereas consumer surplus maximization requires \( \Pi(p_x, p_y) = 0 \).

2. The First-Best Optimum

A consumer who decides to purchase the product should choose a use level, \( y \), that solves: \( \max_y (u(y) - yc_y) \). The FOC, \( u'(y^*) = c_y \), implicitly defines the optimal use level as: \( y^* = y(c_y) \). A consumer should choose to purchase the product iff \( v > v^* = v(c_x, c_y) = (c_x + y(c_y)c_y) - u(y(c_y)) \). The product should be purchased by the following number of consumers: \( D^*(c_x, c_y) = 1 - F(v(c_x, c_y)) \).

Therefore, social welfare at the first-best optimum is:

\[
W^* = W(c_x, c_y; 1) = \int_{\phi(c_x, c_y)}^{\infty} \left[ v + u(y(c_y)) - (c_x + y(c_y)c_y) \right] f(v) dv
\]
E. The Law

I study the effects of a rule that restricts the permissible magnitude of either $p_y$ or $p_x$. Specifically, I consider a price cap, $\tilde{p}_y$, that adds a “legal constraint” $p_y \leq \tilde{p}_y$ to the seller’s optimization problem; and a price cap, $\tilde{p}_x$, that adds a “legal constraint” $p_x \leq \tilde{p}_x$ to the seller’s optimization problem. The question is under what conditions will such a rule increase social welfare and under what conditions will it decrease social welfare.

The analysis proceeds as follows. I begin, in Section 3, by deriving the second-best optimum that can be attained given a certain level of misperception ($\delta$) – both for utility misperception and for price misperception. Then I derive the equilibrium outcomes and welfare levels and compare them to the second-best benchmark. The analysis depends on market structure. The perfect competition case is considered in Section 4. The monopoly case is considered in Section 5.

3. The Second-Best Optimum

A. General

The benchmark for the welfare analysis is the second-best optimum. I consider two normative criteria: maximization of the (actual) consumer surplus and social welfare maximization. The second-best optimum is the maximum consumer surplus or maximum welfare level that can be attained given a certain level of misperception ($\delta$). This second-best optimum is attained by setting prices, $p^*_y(\delta)$ and $p^*_x(\delta)$, to maximize $W(p_x, p_y; \delta) = S(p_x, p_y; \delta)$ s.t. the seller’s participation constraint, which is $\Pi(p_x, p_y; \delta) = 0$ when we
are maximizing consumer surplus and $\Pi(p_x, p_y; \delta) \geq 0$ when we are maximizing social welfare.

With utility overestimation and price underestimation, demand is too high. When we are maximizing social welfare, the second-best response to excessive demand is to increase $p_x$ beyond the first-best level, i.e., set $p_x > c_x$, and keep $p_y = c_y$ in order to avoid distortions in the use decision. This response achieves the first-best welfare level.

When we are maximizing consumer surplus, the second-best response is different. Raising $p_x$ (and leaving $p_y$ unchanged) violates the zero profit constraint, $\Pi(p_x, p_y; \delta) = 0$. Intuitively, we can obtain a larger consumer surplus by implementing a smaller increase in the total price, albeit at the cost of setting $p_y \neq c_y$ and distorting use decisions. Specifically, with utility overestimation, consumer surplus is maximized by setting $p_y > c_y$ and $p_x < c_x$ (to satisfy the zero-profit constraint). Since utility is overestimated and, as a result, use is overestimated, raising $p_y$ has a disproportionate effect on demand: We can most effectively combat the excessive demand problem by raising $p_y$. With price underestimation, consumer surplus is maximized by setting $p_y < c_y$ and $p_x > c_x$ (to satisfy the zero-profit constraint). We can most effectively combat the excessive demand problem by shifting pricing away from the underestimated dimension.

With utility underestimation and price overestimation, demand is too low. This problem cannot be easily solved with a “costless” adjustment of the base price, even when we are maximizing social welfare. The adjustment would have to be a reduction in $p_x$ below the first-best level, i.e., setting $p_x < c_x$, and this would violate the non-negative profit constraint $\Pi(p_x, p_y; \delta) \geq 0$ (if we keep $p_y = c_y$). With utility underestimation and
price overestimation, the non-negative profit constraint is binding and, therefore, the second-best optimal prices are the same, regardless of whether we are maximizing social welfare or consumer surplus. Specifically, with utility underestimation, a reduction in $p_x$ below the first-best level, i.e., setting $p_x < c_x$, is optimal, but it will be accompanied by an increase in the per-use price, such that $p_y > c_y$. With price overestimation, the most effective way to combat the low demand problem is by shifting pricing away from the overestimated dimension, namely, by reducing the per-use price, such that $p_y < c_y$, and increasing the base price, such that $p_x > c_x$.

To summarize, with utility overestimation and price underestimation the first-best social welfare level can be achieved, despite the misperception. In these cases, second-best prices are different for social welfare maximization and for consumer surplus maximization. With utility underestimation and price overestimation, the two maximization problems converge.

To proceed beyond these general observations, we must turn to a separate analysis of utility misperception and price misperception.

B. Utility Misperception

I begin with utility misperception. The results are summarized in Lemma 1.

*Lemma 1 (Second-Best Optimum, Utility Misperception):*

(a) *When use value is underestimated, i.e., when $\delta < 1$: The second-best per-use price, $p_y$, satisfies $p_y(\delta) > c_y$, and the second-best base price, $p_x$, satisfies $p_x(\delta) < c_x$.*

Social welfare is reduced by the misperception. Given the misperception, optimal pricing
balances the social cost of distorted use levels (generated by \( p_y^*(\delta) > c_y \)) against the social benefit from reducing distortions in purchase levels.

(b) When use value is overestimated, i.e., when \( \delta > 1 \): When we are maximizing consumer surplus, the second-best per-use price, \( p_y \), satisfies: \( p_y^*(\delta) > c_y \), and the second-best base price, \( p_x \), satisfies: \( p_x^*(\delta) < c_x \). Social welfare is reduced by the misperception. Given the misperception, optimal pricing balances the social cost of distorted use levels (generated by \( p_y^*(\delta) > c_y \)) against the social benefit from reducing distortions in purchase levels. When we are maximizing social welfare, the second-best per-use price, \( p_y \), satisfies: \( p_y^*(\delta) = c_y \), and the second-best base price, \( p_x \), satisfies: \( p_x^*(\delta) > c_x \). Social welfare is reduced by the misperception, but optimal pricing restores the first-best welfare level.

Proof: See Appendix.

Remark:

Given the misperception, cost-based pricing, i.e., \( p_y = c_y \) and \( p_x = c_x \), is no longer optimal. Rather it is optimal to increase \( p_y \) (above \( c_y \)) and reduce \( p_x \) (below \( c_x \)), when we are maximizing consumer surplus. This pricing pattern holds for both under- and overestimation of use values, but for different reasons:

(a) Underestimation (\( \delta < 1 \)): Consider an increase in \( p_y \) (above \( c_y \)). This increase has two effects: (1) it decreases the actual value of the product, as the higher per-use price leads to inefficiently low use levels; and (2) it increases the perceived value of the product, since the increase in \( p_y \) and decrease in \( p_x \) counteract the reduction in
demand caused by the misperception. Because of Effect (1), a higher \( p_y \) (and lower \( p_x \)) is bad for the infra-marginal consumers who would have purchased even when \( p_y = c_y \) (and \( p_x = c_x \)). Because of Effect (2), a higher \( p_y \) (and lower \( p_x \)) is good for the marginal consumers who now purchase and gain positive actual value. Since Effect (1) is zero when \( p_y = c_y \) (and \( p_x = c_x \)), the second-best prices satisfy: \( p_y^*(\delta) > c_y \) and \( p_x^*(\delta) < c_x \).

(b) Overestimation (\( \delta > 1 \)): The difference between over- and underestimation is in Effect (2). With overestimation, the first-order effect of the misperception is to artificially inflate demand. Second-best optimal pricing thus works to reduce demand. When use is overestimated, and thus the effect of \( p_y \) is overestimated, an increase in \( p_y \), and decrease in \( p_x \), achieves this demand-reduction goal.

When we are maximizing social welfare, the second-best response to the excessive demand caused by utility overestimation is to increase \( p_x \) beyond the first-best level, i.e., set \( p_x > c_x \), and keep \( p_y = c_y \) in order to avoid distortions in the use decision. This response achieves the first-best welfare level.

C. Price Misperception

Next consider price misperception. To keep the analysis simpler, I introduce the following assumption:

*Assumption 2:* With price misperception, \( \delta \bar{y}(p_y; \delta) \) is monotonically increasing in \( \delta \) for all \( p_y \).
Note that, with price misperception, a higher $\delta$ reduces the perceived use-level $\delta y(p_y; \delta)$ and so without Assumption 2 the effect of $\delta$ on $\delta y(p_y; \delta)$ would be ambiguous.

We can now characterize the second-best optimum. The results are summarized in Lemma 2.

**Lemma 2 (Second-Best Optimum, Price Misperception):**

(a) When the per-use price is underestimated, i.e., when $\delta < 1$: When we are maximizing consumer surplus, the second-best per-use price, $p_y$, satisfies: $p_y^*(\delta) < c_y$, and the second-best base price, $p_x$, satisfies: $p_x^*(\delta) > c_x$. Social welfare is reduced by the misperception. Given the misperception, optimal pricing balances the social cost of distorted use levels (generated by $p_y^*(\delta) < c_y$) against the social benefit from reducing distortions in purchase levels. When we are maximizing social welfare, the second-best per-use price, $p_y$, satisfies: $p_y^*(\delta) = c_y$, and the second-best base price, $p_x$, satisfies: $p_x^*(\delta) > c_x$. Social welfare is reduced by the misperception, but optimal pricing restores the first-best welfare level.

(b) When the per-use price is overestimated, i.e., when $\delta > 1$: The second-best per-use price, $p_y$, satisfies: $p_y^*(\delta) < c_y$, and the second-best base price, $p_x$, satisfies: $p_x^*(\delta) > c_x$. Social welfare is reduced by the misperception. Given the misperception, optimal pricing balances the social cost of distorted use levels (generated by $p_y^*(\delta) < c_y$) against the social benefit from reducing distortions in purchase levels.

Proof: See Appendix.
Remark:

With price misperception, as with utility misperception, cost-based pricing, i.e., \( p_y = c_y \) and \( p_x = c_x \), is no longer optimal. The second-best optimal prices, however, are very different with price misperception. Specifically, when we are maximizing consumer surplus, it is optimal to reduce \( p_y \) (below \( c_y \)) and increase \( p_x \) (below \( c_x \)). This pricing pattern holds for both under- and overestimation of the per-use price:

(a) Underestimation \((\delta < 1)\): While utility underestimation results in inadequately low demand, price underestimation results in excessive demand. To counter-act this first-order effect of the misperception, the second-best optimum requires a decrease in the misperceived price, \( p_y \), and a corresponding increase in the accurately perceived price, \( p_x \). This price adjustment reduces the adverse effect of the misperception.

(b) Overestimation \((\delta > 1)\): While utility overestimation results in excessive demand, price overestimation results in inadequately low demand. To counter-act this first-order effect of the misperception, the second-best optimum requires a decrease in the misperceived price, \( p_y \), and a corresponding increase in the accurately perceived price, \( p_x \).

When we are maximizing consumer surplus, with both under- and overestimation it is second-best optimal to shift pricing away from the misperceived price dimension and to the accurately perceived price dimension.

When we are maximizing social welfare, the second-best response to the excessive demand caused by price underestimation is to increase \( p_x \) beyond the first-best level, i.e.,
set $p_x > c_x$, and keep $p_y = c_y$ in order to avoid distortions in the use decision. This response achieves the first-best welfare level.

D. Comparison

When we are maximizing consumer surplus, the second-best optimum pricing structure critically depends on the type of misperception. With utility misperception, the optimal response to both under- and overestimation entails a high per-use price and a low base price. Conversely, with price misperception, the optimal response to both under- and overestimation entails a low per-use price and a high base price.

When we are maximizing social welfare, the key question is whether we are responding to excessive demand or to inadequately low demand. With utility overestimation and price underestimation, the problem is excessive demand. This problem can be solved by raising the base price, such that the first-best welfare level is maintained. With utility underestimation and price overestimation, the problem is inadequately low demand. Here there is no “easy” fix. Second-best optimal prices are identical to those that maximize consumer surplus.

4. Competition

A. General

In a competitive market, sellers set prices to maximize the perceived consumer surplus, subject to a break-even constraint. Formally, the seller solves the following maximization problem:

$$\max_{p_x, p_y} S(p_x, p_y; \delta) \text{ s.t. } \Pi(p_x, p_y) = 0$$
Note that $\Pi(p_x, p_y) = (p_x - c_x + y(p_y) \cdot (p_y - c_y)) \cdot \left[1 - F\left(\varphi(p_x, p_y; \delta)\right)\right]$, and so the zero-profit constraint implies: $p_x = c_x - y(p_y) \cdot (p_y - c_y)$.

Starting with the per-use price, I take the derivative of the perceived consumer surplus, $\hat{S}(p_x, p_y; \delta)$, w.r.t. $p_y$:

$$\frac{\partial \hat{S}}{\partial p_y} = - \frac{\partial \varphi(p_x, p_y; \delta)}{\partial p_y} \cdot \hat{V}(\varphi(p_x, p_y; \delta), p_x, p_y; \delta) f \left(\varphi(p_x, p_y; \delta)\right)$$

$$\qquad \qquad + \int_{\varphi(p_x, p_y; \delta)}^{\infty} \left[ \frac{\partial \hat{V}(v, p_x, p_y; \delta)}{\partial p_y} \right] f(v) dv$$

Since $\hat{V}(\varphi(p_x, p_y; \delta), p_x, p_y; \delta) = 0$ (by definition), the marginal effect is zero: For the marginal consumer, the perceived value from purchasing the product is zero. We are thus left with the infra-marginal effect:

$$\frac{\partial \hat{S}}{\partial p_y} = \int_{\varphi(p_x, p_y; \delta)}^{\infty} \left[ \frac{\partial \hat{V}(v, p_x, p_y; \delta)}{\partial p_y} \right] f(v) dv$$

Next, consider social welfare: To find the competitive equilibrium, I maximized

$$\hat{S}(p_x, p_y; \delta) = \int_{\varphi(p_x, p_y; \delta)}^{\infty} \hat{V}(v, p_x, p_y; \delta) f(v) dv$$

s.t. $\Pi(p_x, p_y) = 0$. The second-best optimum (see Section 3 above) was derived by maximizing

$$W(p_x, p_y; \delta) = \int_{\varphi(p_x, p_y; \delta)}^{\infty} \left[ v + u \left(y(p_y)\right) - (c_x + y(p_y)c_y) \right] f(v) dv$$

s.t. $\Pi(p_x, p_y) = 0$. Sellers in a competitive market care about maximizing the perceived value to their customers, not about maximizing actual surplus. This is the source of the inefficiency.
Note that misperception is the underlying problem. Without misperception, i.e., when \( \delta = 1 \), the per-use price, \( p_y \), is: \( p_y^c = c_y \), the base price, \( p_x \), is: \( p_x^c = c_x \), and the first-best socially optimal welfare level is obtained. These are simply the standard results regarding the efficiency properties of perfect competition, under the (standard) assumption that consumers do not suffer from any misperception. I next study the effects of consumer misperception.

B. Utility Misperception

Using the Envelope Theorem, the derivative \( \frac{d\mathcal{S}}{dp_y} \) can be rewritten as:

\[
\frac{d\mathcal{S}}{dp_y} = - \left[ \frac{\partial (p_y; \delta)}{\partial p_y} + \frac{dp_x}{dp_y} \right] \left[ 1 - F \left( \vartheta(p_x, p_y; \delta) \right) \right]
\]

As noted above, this derivative represents the infra-marginal effect: A $1 increase in \( p_y \) raises the total price that the consumer expects to pay for the use dimension by \( \frac{\partial (p_y; \delta)}{\partial p_y} \). The zero-profit constraint implies that the increase in \( p_y \) will also result in a reduction in \( p_x \): \( \frac{dp_x}{dp_y} = 0 \). Using the zero-profit constraint to find \( \frac{dp_x}{dp_y} \), the FOC becomes:

\[
(1) \quad p_y = \frac{c_y}{\partial_y} + \left[ \frac{\vartheta(p_y; \delta)}{\partial (p_y)} - \vartheta(p_y) \right] \frac{dy(p_y)}{dp_y}
\]

Or:

\[
(1a) \quad 1 + \eta_{y,p_y} \left( \frac{p_y-c_y}{p_y} \right) \cdot y(p_y) = \frac{\vartheta(p_y; \delta)}{\partial (p_y)}
\]

\[2\] The increase in \( p_y \) also reduces the expected use level, \( \vartheta(p_y; \delta) \), which in turn reduces the expected utility from the use dimension and the total price that the consumer expects to pay for the use dimension. But, since the expected use level is optimally set to solve \( \max_y (\delta u(y) - y p_y) \), these two effects cancel out (the Envelope Theorem).
from which the competitive per-use price, \( p^C_y \), can be derived. The base price, \( p^C_x \), can be derived from the zero profit constraint, \( \Pi(p_x, p_y) = 0 \):

\[
(2) \quad p_x = c_x - y(p_y) \cdot (p_y - c_y)
\]

Lemma 3 describes equilibrium outcomes in a competitive market with utility misperception and evaluates their welfare implications.

**Lemma 3 (Competitive Equilibrium, Utility Misperception):** In a competitive market –

*(a)* When use value is underestimated, i.e., when \( \delta < 1 \): The per-use price, \( p_y \), satisfies: \( p^C_y(\delta) > p^*_y(\delta) > c_y \), and the base price, \( p_x \), satisfies: \( p^C_x(\delta) < p^*_x(\delta) < c_x \). Social welfare is reduced by the misperception. Given the misperception, equilibrium prices deviate from the second-best optimal prices, distorting use levels and inducing excessive demand.

*(b)* When use value is overestimated, i.e., when \( \delta > 1 \): When we are maximizing consumer surplus, the per-use price, \( p_y \), satisfies: \( p^C_y(\delta) < c_y < p^*_y(\delta) \), and the base price, \( p_x \), satisfies: \( p^C_x(\delta) > c_x > p^*_x(\delta) \). Social welfare is reduced by the misperception. Given the misperception, equilibrium prices deviate from the second-best optimal prices, indeed they move in the wrong direction vis-à-vis cost, distorting use levels and inducing excessive demand. When we are maximizing social welfare, the per-use price, \( p_y \), satisfies: \( p^C_y(\delta) < c_y = p^*_y(\delta) \), and the base price, \( p_x \), satisfies: \( p^C_x(\delta) > p^*_x(\delta) > c_x \).

Proof: See Appendix.
Remark:

(a) Utility Underestimation:

*The per-use price.* If consumers underestimate use-values, they will also underestimate use levels and the importance of the per-use price, $p_y$. Sellers, responding to this misperception, will increase $p_y$ above the efficient level. Second-best optimal price adjustment, in response to the misperception, results in $p_y^*(\delta) > c_y$ and $p_x^*(\delta) < c_x$ (see Lemma 1). These second-best prices balance the two effects of an increase in $p_y$ (above $c_y$): (1) A decrease in the actual value of the product, as the higher per-use price leads to inefficiently low use levels; and (2) An increase in the perceived value of the product, as the increase in $p_y$ and decrease in $p_x$ counteract the reduction in demand caused by the misperception. Sellers care only about increasing demand (Effect (1)) and not about the actual value provided to consumers (Effect (2)). Therefore, equilibrium prices will deviate from the second-best prices. Specifically, $p_y$ will be set too high.

*The base price.* The zero-profit condition implies that if $p_y$ is set above cost, such that sellers are making positive profits on the use dimension, then $p_x$ must be set below cost. Sellers increase $p_y$ to extract more revenue from the use dimension, since this allows them to extract money from consumers while incurring a smaller cost in terms of lost demand (because of the misperception). In a competitive market, if sellers extract more revenue on the use dimension (the Y dimension), they must extract less through the base price, or incur a bigger loss on the X dimension. Therefore, an increase in $p_y$ above the second-best optimal level will be accompanied by a decrease in $p_x$ below the second-best optimal level.
**Welfare.** The excessively high $p_y$ will distort use-level decisions (beyond what is optimal, given the misperception). In addition, the excessively high $p_y$, and corresponding $p_x$, will inefficiently inflate demand, such that the marginal gain from more purchases is outweighed by the infra-marginal loss in the value of each purchase.

(b) Utility Overestimation: If consumers overestimate use-values, they will also overestimate use levels and the importance of the per-use price, $p_y$. Sellers, responding to this misperception, will decrease $p_y$ below $c_y$ (Recall, from Lemma 1, that second-best efficiency requires $p_y$ above $c_y$.) When $p_y < c_y$, the zero-profit condition implies $p_x > c_x$. (Recall, from Lemma 1, that second-best efficiency requires $p_x$ below $c_x$.) With utility overestimation, equilibrium prices go in opposite direction, vis-à-vis cost, from the second-best prices. While second-best prices work to mitigate the effect of the misperception, equilibrium prices exacerbate the effect of the misperception. The result is inefficiently inflated demand. In addition, the deviations from cost-based pricing distorts use-level decisions.

We can now study the effects of imposing a price cap $\bar{p}_y$ or $\bar{p}_x$, namely of adding a legal constraint $p_y \leq \bar{p}_y$ or $p_x \leq \bar{p}_x$. We first note the standard result that, without misperception, a price cap can only reduce social welfare. In our model, it is the misperception that generates the welfare costs and opens the door for potentially welfare-enhancing regulation. In particular, when consumers underestimate the use value, i.e., when $\delta < 1$, the per-use price, $p_y$, will be excessively high without legal intervention (see Lemma 3), and so a price cap, $\bar{p}_y$, can increase social welfare (as long as the cap is not set too low). When consumers overestimate the use value, i.e., when $\delta > 1$, the base
price, \( p_x \), will be excessively high without legal intervention (see Lemma 3), and so a properly calibrated price cap, \( \bar{p}_x \), can increase social welfare.

These results are summarized in Proposition 1.

**Proposition 1 (Competition, Utility Misperception):** In a competitive market –

(a) When use value is underestimated, i.e., when \( \delta < 1 \), a mild, though still binding, price cap, \( \bar{p}_y \), satisfying \( p_y^C(\delta) > \bar{p}_y \geq p_y^*(\delta) \geq c_y \), reduces the per-use price, \( p_y \), and increases the base price, \( p_x \), resulting in increased social welfare.

(b) When use value is overestimated, i.e., when \( \delta > 1 \), a mild, though still binding, price cap, \( \bar{p}_x \), satisfying \( p_x^C(\delta) > \bar{p}_x \geq p_x^*(\delta) \), reduces the base price, \( p_x \), and increases the per-use price, \( p_y \), resulting in increased social welfare.

Remark: The results stated in Proposition 1 follow from Lemma 3.

(a) Utility Underestimation: A mild, yet binding legal constraint brings the per-use price closer to the second-best optimal price, increasing social welfare. The reduction in \( p_y \) is accompanied by a corresponding increase in \( p_x \). Note, however, that a strict constraint, \( \bar{p}_y < p_y^*(\delta) \), can reduce welfare. A strict constraint reduces the per-use price below the second-best optimal price. The base price, \( p_x \), increases in response. This can either increase or decrease social welfare, as we move from an excessively high price to an inadequately low price.

(b) Utility Overestimation: The relevant cap here is on the base price, \( p_x \). The second-best optimal base price, \( p_x^*(\delta) \), is below cost, while the equilibrium base price (without legal intervention) is above cost. Accordingly, there is more room for a welfare-
enhancing price cap. And, still, the lawmaker should take care not to set \( \bar{p}_x < p^*_x(\delta) \), since such a strict cap might reduce welfare.

C. Price Misperception

The analysis with price misperception turns out to be very similar to the analysis with utility misperception. The only change in the FOCs is that \( \bar{y}(p_y; \delta) \) needs to be replaced with \( \delta \bar{y}(p_y; \delta) \). Accordingly, the FOC w.r.t. \( p_y \) is:

\[
(3) \quad p_y = c_y + \left[ \delta \bar{y}(p_y; \delta) - y(p_y) \right] \frac{dy(p_y)}{dp_y}
\]

Or:

\[
(3a) \quad \left[ 1 + \eta_{y,p_y} \left( \frac{p_y - c_y}{p_y} \right) \right] \cdot y(p_y) = \delta \bar{y}(p_y; \delta)
\]

from which the competitive per-use price, \( p^C_y \), can be derived. Equation (2), characterizing the base price, \( p^C_x \), remains unchanged when we move from utility misperception to price misperception.

Lemma 4 describes equilibrium outcomes in a competitive market with price misperception and evaluates their welfare implications.

Lemma 4 (Competitive Equilibrium, Price Misperception): In a competitive market –

(a) When the per-use price is underestimated, i.e., when \( \delta < 1 \): When we are maximizing consumer surplus, the per-use price, \( p_y \), satisfies: \( p^C_y(\delta) > c_y > p^*_y(\delta) \), and the base price, \( p_x \), satisfies: \( p^C_x(\delta) < c_x < p^*_x(\delta) \). Social welfare is reduced by the misperception. Given the misperception, equilibrium prices deviate from the second-best optimal prices, indeed they move in the wrong direction vis-à-vis cost, distorting
use levels and inducing excessive demand. When we are maximizing social welfare, the per-use price, \( p_y \), satisfies: \( p_y^C(\delta) > c_y = p_y^*(\delta) \), and the base price, \( p_x \), satisfies: \( p_x^C(\delta) < p_x^*(\delta) < c_x \).

(b) When the per-use price is overestimated, i.e., when \( \delta > 1 \): The per-use price, \( p_y \), satisfies: \( p_y^C(\delta) < p_y^*(\delta) < c_y \), and the base price, \( p_x \), satisfies: \( p_x^C(\delta) > p_x^*(\delta) > c_x \). Social welfare is reduced by the misperception. Given the misperception, equilibrium prices deviate from the second-best optimal prices, distorting use levels and inducing excessive demand.

Proof: Omitted.

Remark:

(a) Price Underestimation: When the per-use price is underestimated, sellers increase this price dimension (and reduce the base price) to exacerbate the effect of the misperception and further inflate demand.

(b) Price Overestimation: When the per-use price is overestimated, sellers shift pricing away from this price dimension, and towards the base price. Here the misperception reduces demand, and the price shift minimizes this demand-reducing effect.

We can now study the effects of imposing a price cap. With (price) underestimation, i.e., when \( \delta < 1 \), the per-use price, \( p_y \), will be excessively high without legal intervention (see Lemma 4), and so a price cap, \( \bar{p}_y \), can increase social welfare. With (price)
overestimation, i.e., when $\delta > 1$, the base price, $p_x$, will be excessively high without legal intervention (see Lemma 4), and so a price cap, $\bar{p}_x$, can increase social welfare.

These results are summarized in Proposition 2.

**Proposition 2 (Competition, Price Misperception):** In a competitive market –

(a) When the per-use price is underestimated, i.e., when $\delta < 1$, a mild, though still binding, price cap, $\bar{p}_y$, satisfying $p_y^c(\delta) > \bar{p}_y \geq p_y^*(\delta)$, reduces the per-use price, $p_y$, and increases the base price, $p_x$, resulting in increased social welfare.

(b) When the per-use price is overestimated, i.e., when $\delta > 1$, a mild, though still binding, price cap, $\bar{p}_x$, satisfying $p_x^c(\delta) > \bar{p}_x \geq p_x^*(\delta) > c_x$, reduces the base price, $p_x$, and increases the per-use price, $p_y$, resulting in increased social welfare.

Remark: The results stated in Proposition 2 follow from Lemma 4.

(a) Price Underestimation: The second-best optimal per-use price, $p_y^*(\delta)$, is below cost, while the equilibrium per-use price (without legal intervention) is above cost. Accordingly, there is more room for a welfare-enhancing price cap.

(b) Price Overestimation: A mild, yet binding legal constraint brings the base price closer to the second-best optimal price, increasing social welfare.

Remark (Political Economy): Policymakers intervene and cap a price when this price dimension, and its adverse implications, become politically salient. If a price dimension that has become politically salient has also become salient to consumers, i.e., the misperception has been cured, then legal intervention will only harm consumers and
reduce welfare. This link between political salience and market salience is more likely with price misperception (Proposition 2), but less likely with utility misperception (Proposition 1).

D. Comparison: The Object (and Direction) of Misperception

To facilitate a comparison between the welfare and policy implications of different types (and directions) of misperception, we collect the results from Lemmas 3 and 4 in the following Table. (When the results for social welfare maximization differ from the results for consumer surplus maximization, they are presented in parentheses.)

<table>
<thead>
<tr>
<th>Misperception Type</th>
<th>Utility Misperception</th>
<th>Price Misperception</th>
</tr>
</thead>
</table>
| Underestimation    | \( p_y^C(\delta) > p_y^*(\delta) > c_y \) | \( p_y^C(\delta) > c_y > p_y^*(\delta) \)  
\( p_x^C(\delta) < p_x^*(\delta) < c_x \)  
\( p_x^C(\delta) < c_x < p_x^*(\delta) \) |
| Overestimation     | \( p_y^C(\delta) < c_y < p_y^*(\delta) \)  
\( p_y^C(\delta) < c_y = p_y^*(\delta) \)  
\( p_x^C(\delta) > c_x > p_x^*(\delta) \)  
\( p_x^C(\delta) > p_x^*(\delta) > c_x \)  
\( p_x^C(\delta) > p_x^*(\delta) > c_x \) |

Table 1: Price Distortions for Different Types (and Directions) of Misperception

*Underestimation.* Equilibrium per-use prices are similarly above cost with both utility and price misperception, but the second-best prices are different and, accordingly,
the welfare implications are different: With utility misperception, the second-best price is above the per-use cost, $c_y$ (or equal to the per-use cost, when we are maximizing social welfare), and the equilibrium price is even higher: $p^C_y(\delta) > p^*_y(\delta) \geq c_y$. With price misperception, the second-best price is below the per-use cost, $c_y$, and the equilibrium price is above the per-use cost: $p^C_y(\delta) > c_y > p^*_y(\delta)$. Accordingly, the cost – in terms of both consumer surplus and social welfare – is larger with price misperception. (While $p^C_y(\delta) > c_y$ for both utility and price misperception, the exact price depends on the type of misperception.)

The optimal scope of legal intervention depends on the welfare cost or cost in term of reduced consumer surplus. A price cap, $\bar{p}_y$, in the range $[p^*_y(\delta), p^C_y(\delta)]$ increases welfare and consumer surplus with certainty. This range, we have now seen, is larger with price misperception. Specifically, with utility misperception a price cap above the per-unit cost, $c_y$, might fall outside the range, and thus reduce welfare and consumer surplus; with price misperception a price cap above the per-unit cost, $c_y$, will always increase welfare.

*Overestimation.* With overestimation, we focus on the base-price, rather than on the per-use price. When we are maximizing consumer surplus, the welfare picture is the mirror image of what we saw for underestimation: The cost in terms of lost consumer surplus, and the scope for consumer-surplus-enhancing price caps, are larger with utility misperception. When we are maximizing social welfare, the welfare cost, and the scope for welfare-enhancing price caps, are similarly small with both utility and price misperception.
Summary. Market outcomes are often sensitive to the type of misperception that consumers suffer from. Here, however, we find that two types of plausible misperceptions – utility misperception and price misperception – have very similar effects on equilibrium prices. Nevertheless, since second-best optimal pricing depends on the type of misperception, welfare consequences (and consequences for consumer surplus) and policy implications also depend on the type of misperception.

These comparisons are depicted graphically in Figure 1 below. Figure 1a depicts the comparisons for consumer surplus maximization, and Figure 1b depicts the comparisons for social welfare maximization.

[Figure 1 goes here. The Figure is provided at the end of this document.]

5. Monopoly

A. General

We next consider a monopolistic market. A monopolistic seller sets prices, \( p_x \) and \( p_y \), to maximize its profit function: \( \Pi(p_x, p_y) = \pi(p_x, p_y) \cdot \left[ 1 - F\left( \varphi(p_x, p_y; \delta) \right) \right] \).

Consider, first, how the monopolist sets the base price, \( p_x \). The derivative of the profit function w.r.t. \( p_x \) is:

\[
\frac{\partial \Pi(p_x, p_y)}{\partial p_x} = \left[ 1 - F\left( \varphi(p_x, p_y) \right) \right] - \pi(p_x, p_y) \cdot f(\varphi(p_x, p_y))
\]

An increase in \( p_x \) has two effects:
1) An infra-marginal effect: It increases the monopolist’s per-customer profit by 
\[ \frac{\partial \pi(p_x, p_y)}{\partial p_x}, \] 
which applies to all \(1 - F\left(\tilde{\sigma}(p_x, p_y)\right)\) customers. Note that \(\frac{\partial \pi(p_x, p_y)}{\partial p_x} = 1\).

2) A marginal effect: It reduces the demand for the monopolist’s product, i.e., it reduces the number of customers, by 
\[ f\left(\tilde{\sigma}(p_x, p_y)\right) \cdot \frac{\partial \tilde{\sigma}(p_x, p_y)}{\partial p_x}, \] 
with the monopolist losing \(\pi(p_x, p_y)\) per-each lost customer. Note that \(\frac{\partial \tilde{\sigma}(p_x, p_y)}{\partial p_x} = 1\).

Next consider how the monopolist sets the per-use price, \(p_y\). The derivative of the profit function w.r.t. \(p_y\) is:
\[
\frac{\partial \Pi(p_x, p_y)}{\partial p_y} = \frac{\partial \pi(p_x, p_y)}{\partial p_y} \cdot \left[1 - F\left(\tilde{\sigma}(p_x, p_y)\right)\right] - \pi(p_x, p_y) \cdot f\left(\tilde{\sigma}(p_x, p_y)\right) \cdot \frac{\partial \tilde{\sigma}(p_x, p_y)}{\partial p_y}
\]

An increase in \(p_y\) has two effects:

1) An infra-marginal effect: It increases the monopolist’s per-customer profit by 
\[ \frac{\partial \pi(p_x, p_y)}{\partial p_y}, \] 
which applies to all \(1 - F\left(\tilde{\sigma}(p_x, p_y)\right)\) customers. Note that \(\frac{\partial \pi(p_x, p_y)}{\partial p_y} = y(p_y) + \frac{dy(p_y)}{dp_y} \cdot (p_y - c_y) = \left[1 + \eta_{y,p_y} \cdot \left(\frac{p_y - c_y}{p_y}\right)\right] \cdot y(p_y) > 0\). The effect on the per-customer profit has two components: (a) a higher price increases the profit per unit used – the marginal increase equals the number of units used, \(y(p_y)\); and (b) a higher price reduces the number of units used by \(\frac{dy(p_y)}{dp_y}\), reducing per-customer profit by \(\frac{dy(p_y)}{dp_y} \cdot (p_y - c_y)\). Assumption 1 implies that \(\frac{\partial \pi(p_x, p_y)}{\partial p_y} > 0\).
2) A marginal effect: It reduces the demand for the monopolist’s product, i.e., it reduces the number of customers, by \( f \left( \varphi(p_x, p_y) \right) \cdot \frac{\partial \varphi(p_x, p_y)}{\partial p_y} \), with the monopolist losing \( \pi(p_x, p_y) \) per each lost customer.

The equilibrium outcome is derived by solving the system of two FOCs, \( \frac{\partial \Pi(p_x, p_y)}{\partial p_x} = 0 \) and \( \frac{\partial \Pi(p_x, p_y)}{\partial p_y} = 0 \).

This model can be used to replicate standard monopoly results, under the (standard) assumption that consumers do not suffer from any misperception. The monopolist would set an efficient per-use price, \( p_y^M = c_y \), to maximize total surplus (a different \( p_y \) distorts use-level decisions and reduces total surplus) and use the base price to extract monopolistic rents. The base price would be set above the competitive level, which leads to an inefficiently small number of products purchased. The result is a welfare loss – the monopoly deadweight loss. Our focus, however, is on the implications – both descriptive and normative – of consumer misperception. We begin with utility misperception, in subsection B and then turn to price misperception in subsection C. Subsection D compares the results across the different types (and directions) of misperception and highlights the implications of market power.

B. Utility Misperception

With utility misperception, we have:

\[
\frac{\partial \varphi(p_x, p_y)}{\partial p_y} = \left( p_y - \delta u' \left( \psi(p_y, \delta) \right) \right) \cdot \frac{d \psi(p_y; \delta)}{dp_y} + \psi(p_y, \delta) = \psi(p_y, \delta)
\]

The derivative of the profit function w.r.t. \( p_y \) can now be written as:
\[
\frac{\partial \Pi(p_x,p_y)}{\partial p_y} = \left[ 1 - F\left( \vartheta(p_x,p_y) \right) \right] \cdot \left[ 1 + \eta_{y,p_y} \cdot \left( \frac{p_y - c_y}{p_y} \right) \right] \cdot y(p_y) + \frac{-f\left( \vartheta(p_x,p_y) \right) \cdot \pi(p_x,p_y) \cdot \hat{y}(p_y, \delta)}{}
\]

Taken together, the two FOCs, \( \frac{\partial \Pi(p_x,p_y)}{\partial p_x} = 0 \) and \( \frac{\partial \Pi(p_x,p_y)}{\partial p_y} = 0 \), imply:

\[
\left[ 1 + \eta_{y,p_y} \cdot \left( \frac{p_y - c_y}{p_y} \right) \right] \cdot y(p_y) = g(p_y, \delta)
\]

which is identical to Equation (1a) from our Competition analysis (in Section 4.B.). We thus have: \( p^M_y = p_y^c \). The per-use price, \( p_y \), is independent of market structure. This result follows from the separability of the X and Y dimensions in this model.

Turning to the base price, \( p_x \), the FOC, \( \frac{\partial \Pi(p_x,p_y)}{\partial p_x} = 0 \), implies:

\[
(3) \quad p_x = c_x - y(p_y)(p_y - c_y) + \frac{1-F(\vartheta(p_x,p_y))}{f(\vartheta(p_x,p_y))}
\]

Compare Equation (3) to Equation (2) from the Competition analysis:

\[
(2) \quad p_x = c_x - y(p_y)(p_y - c_y)
\]

Not surprisingly, the base price will be set higher in a monopolistic market.

Lemma 5 describes equilibrium outcomes in a monopolistic market with utility misperception and evaluates their welfare implications.

**Lemma 5 (Monopolistic Market, Utility Misperception):** In a monopolistic market –

(a) When use value is underestimated, i.e., when \( \delta < 1 \):

i. The per-use price, \( p_y \), satisfies: \( p^M_y(\delta) > p^*_y(\delta) > c_y \).

ii. The base price, \( p_x \), can be smaller than, equal to, or greater than \( c_x \). The base price can be smaller than, equal to, or greater than \( p^*_x(\delta) \).
iii. Social welfare is reduced by the misperception. Since $p^M_y(\delta) = p^C_y(\delta)$, the welfare cost can be divided into two components: (1) the cost induced by the misperception (which is identical to the welfare cost under Competition):

$$W \left( p^*_y(\delta), p^*_x(\delta) \right) - W \left( p^M_y(\delta) = p^C_y(\delta), p^C_x(\delta) \right);$$

and (2) the cost induced by the higher monopoly base price:

$$W \left( p^M_y(\delta) = p^C_y(\delta), p^C_x(\delta) \right) - W \left( p^M_y(\delta), p^M_x(\delta) \right).$$

(b) When use value is overestimated, i.e., when $\delta > 1$:

i. The per-use price, $p_y$, satisfies: $p^M_y(\delta) < c_y < p^*_y(\delta)$.

ii. The base price, $p_x$, satisfies: $p^M_x(\delta) > c_x > p^*_x(\delta)$.

iii. Social welfare is reduced by the misperception. Since $p^M_y(\delta) = p^C_y(\delta)$, the welfare effect can be divided into two components: (1) the cost induced by the misperception (which is identical to the welfare cost under Competition):

$$W \left( p^*_y(\delta), p^*_x(\delta) \right) - W \left( p^M_y(\delta) = p^C_y(\delta), p^C_x(\delta) \right);$$

and (2) the welfare effect of the higher monopoly base price:

$$W \left( p^M_y(\delta) = p^C_y(\delta), p^C_x(\delta) \right) - W \left( p^M_y(\delta), p^M_x(\delta) \right),$$

which can be either positive or negative.

Proof: See Appendix.
Remark:

(a) Utility Underestimation:

*The per-use price.* As in the perfect competition case, misperception leads to an inefficiently high per-use price, $p_y^M(\delta) = p_y^C(\delta)$.

*The base price.* In the perfect competition case, the high, above-cost per-use price implied a low, below-cost base price. In the monopoly case, the base price is higher than in the competition case. The magnitude of this difference between the monopoly base price and the competition base price determines the relationship between the monopoly base price and both cost ($c_x$) and the second-best base price ($p_x^* (\delta)$).

*Welfare.* The welfare costs under Monopoly are equal to the welfare costs under Competition, plus the welfare effect of the higher monopoly base price. The higher base price reduces demand. Since, with utility underestimation, the lost purchases generated a positive surplus (the actual utility from a purchase exceeds the perceived utility which determines demand), the reduced demand implies an additional welfare loss. This additional welfare loss can be thought of as the monopolist deadweight loss.

(b) Utility Overestimation: As in the perfect competition case, misperception leads to an inefficiently low per-use price, $p_y$. Moreover, whereas the second-best per-use price is above cost, the equilibrium price is below cost. The monopolist compensates for the low per-use price, which implies a loss on the use dimension, with a higher base price. The monopoly base price is higher than the competitive base price, and it reduces demand below the competitive level. This, however, can be a good thing, because, with overestimation, at least some of these lost purchases would have generated a net social loss.
We can now study the effects of imposing a price cap. Begin with the no
misperception case and consider, first, a cap \( \bar{p}_y \) on the per-use price. A price cap \( \bar{p}_y \geq c_y \) has no effect, since the constraint is not binding. A price cap, \( \bar{p}_y < c_y \), distorts prices and reduces welfare. Specifically, the reduced \( p_y \) leads to excessive use-levels. The monopolist will increase the base price, \( p_x \), to compensate for losses on the use dimension.\(^3\) Next, consider a cap \( \bar{p}_x \) on the base price. Note that in a standard monopoly model, with a single price dimension, a price cap can reduce the monopoly deadweight loss and thus increase welfare. In our two-dimensional price model, a cap on the base price can similarly reduce the monopoly deadweight loss. But the cap will also result in a use-level distortion, as the monopolist increases \( p_y \) in response to the cap on \( p_x \).\(^4\) The overall welfare effect of a cap on \( p_x \) is, therefore, ambiguous.

Our focus, however, is on the effect of a price cap, given consumer misperception. Without legal intervention, underestimation of use value, i.e., \( \delta < 1 \), results in an excessively high \( p_y \) and in a correspondingly low use-level. This distortion leads to a welfare loss. A price cap, \( \bar{p}_y \), will reduce the use-level distortion (as long as the cap is not set too low). The effects of a price cap, \( \bar{p}_y \), on the number of products purchased depends on how a price cap on \( p_y \) affects the monopolist’s choice of \( p_x \). The monopolist realizes

\(^3\) Consider Equation (3) above. With a binding cap that imposes \( p_y < c_y \), we have \(-y(p_y)(p_y - c_y) > 0\), which pushes \( p_x \) up. The reduction in \( p_y \) will also increase demand, \( 1 - F(p_x, p_y) \), raising \( p_x \) further.

\(^4\) With a binding cap on \( p_x \), we have: 
\[
\frac{\partial \pi(p_x, p_y)}{\partial p_x} = \left[ 1 - F(p_x, p_y) \right] \cdot \left[ 1 + \eta_{y, p_y} \cdot \left( \frac{p_y - c_y}{p_y} \right) \right] \cdot y(p_y) - f(p_x, p_y) \cdot \pi(p_x, p_y) \cdot \gamma(p_y) = 0.
\]
Given \( \frac{\partial \pi(p_x, p_y)}{\partial p_x} > 0 \), this FOC implies: \( p_y > c_y \) (since, by Assumption 1, we have: 
\[
\frac{\partial \pi(p_x, p_y)}{\partial p_y} = \left[ 1 + \eta_{y, p_y} \cdot \left( \frac{p_y - c_y}{p_y} \right) \right] \cdot y(p_y) > 0.
\]
that the effect, on demand, of a higher $p_x$, is larger than the effect of a higher $p_y$, which is muted by the misperception. Accordingly, the monopolist will hesitate to compensate for the lower $p_y$ by increasing $p_x$. As a result, the price cap can also reduce the monopoly deadweight loss. In sum, a price cap can increase overall welfare. When consumers overestimate the use value, i.e., when $\delta > 1$, it is the base price that is set too high, and so a cap on $p_x$ can increase welfare, if properly calibrated.

These results are summarized in Proposition 3.

**Proposition 3 (Monopoly, Utility Misperception):** In a monopolistic market –

(a) When use value is underestimated, i.e., when $\delta < 1$, a mild, though still binding, price cap, $\bar{p}_y$, satisfying $p_y^M(\delta) \geq \bar{p}_y \geq p_y^*(\delta) \geq c_y$, reduces the per-use price, $p_y$, and increases the base price, $p_x$, resulting in increased social welfare.

(b) When use value is overestimated, i.e., when $\delta > 1$, the higher monopoly base price can either increase or decrease social welfare. Accordingly, a price cap, $\bar{p}_x$, can either decrease or increase social welfare.

Remark: The results stated in Proposition 3 follow from Lemma 5. These results, for a monopolistic market, largely mirror the results stated in Proposition 1, for a competitive market.

C. Price Misperception

As in the Competition case, the analysis with price misperception is very similar to the analysis with utility misperception. Basically, we only need to replace $\frac{\partial \theta(p_x, p_y)}{\partial p_y} =$
\( y(p_y, \delta) \) with \( \frac{\partial y(p_x, p_y)}{\partial p_y} = \delta y(p_y, \delta) \). The equilibrium per-use price is identical to the equilibrium per-use price derived in the competition case for price misperception. The equilibrium base price is defined by Equation (3), as in the monopoly case with utility misperception (and is larger than the base price derived in the competition case for price misperception).

Lemma 6 describes equilibrium outcomes in a monopolistic market with price misperception and evaluates their welfare implications.

Lemma 6 (Monopolistic Market, Price Misperception): In a monopolistic market –

(a) When the per-use price is underestimated, i.e., when \( \delta < 1 \):

i. The per-use price, \( p_y \), satisfies: \( p_y^M(\delta) > c_y > p_y^*(\delta) \).

ii. The base price, \( p_x \), can be smaller than, equal to, or greater than \( c_x \). The base price can be smaller than, equal to, or greater than \( p_x^*(\delta) \).

iii. Social welfare is reduced by the misperception. Since \( p_y^M(\delta) = p_y^C(\delta) \), the welfare effect can be divided into two components: (1) the cost induced by the misperception (which is identical to the welfare cost under Competition):

\[
W \left( p_y^*(\delta), p_x^*(\delta) \right) - W \left( p_y^M(\delta) = p_y^C(\delta), p_x^C(\delta) \right)
\]

and (2) the welfare effect of the higher monopoly base price,

\[
W \left( p_y^M(\delta) = p_y^C(\delta), p_x^C(\delta) \right) - W \left( p_y^M(\delta), p_x^M(\delta) \right),
\]

which can be either positive or negative.

(b) When the per-use price is overestimated, i.e., when \( \delta > 1 \):

i. The per-use price, \( p_y \), satisfies: \( p_y^M(\delta) < p_y^*(\delta) < c_y \).
ii. The base price, $p_x$, satisfies: $p_x^M(\delta) > p_x^C(\delta) > c_x$.

iii. Social welfare is reduced by the misperception. Since $p_x^M(\delta) = p_x^C(\delta)$, the welfare cost can be divided into two components: (1) the cost induced by the misperception (which is identical to the welfare cost under Competition):

$$W \left( p_x^C(\delta), p_x^C(\delta) \right) - W \left( p_x^M(\delta), p_x^C(\delta) \right)$$

and (2) the cost induced by the higher monopoly base price:

$$W \left( p_x^M(\delta), p_x^C(\delta) \right) - W \left( p_x^M(\delta), p_x^M(\delta) \right)$$

Proof: Omitted.

Remark:

As in the competition case, the welfare cost is larger with price underestimation, as compared to utility underestimation. And, conversely, the welfare cost is smaller with price overestimation, as compared to utility overestimation.

As with utility misperception, the monopoly base price is higher than the competitive base price, and it reduces demand below the competitive level. When price is overestimated, this demand reduction effect reduces welfare. However, when price is underestimated, the reduced demand can increase welfare, since at least some of these lost purchases would have generated a net social loss. (With utility misperception, we obtained the opposite results, namely, that the lower demand is welfare reducing for utility underestimation, but can be welfare enhancing for utility overestimation.)
We can now study the effects of imposing a price cap. These effects are summarized in Proposition 4.

**Proposition 4 (Monopoly, Price Misperception):** In a monopolistic market –

(a) When the per-use price is underestimated, i.e., when $\delta < 1$, a mild, though still binding, price cap, $\bar{p}_y$, satisfying $p^M_y(\delta) > \bar{p}_y \geq p^*_y(\delta)$, reduces the per-use price, $p_y$, and increases the base price, $p_x$. The induced repricing reduces demand, and can either increase or decrease social welfare.

(b) When the per-use price is overestimated, i.e., when $\delta > 1$, a mild, though still binding, price cap, $\bar{p}_x$, satisfying $p^M_x(\delta) > \bar{p}_x \geq p^*_x(\delta) > c_x$, reduces the base price, $p_x$, and increases the per-use price, $p_y$, resulting in increased social welfare.

Remark: The results stated in Proposition 4 follow from Lemma 6. These results, for a monopolistic market, largely mirror the results stated in Proposition 3, for a competitive market.

D. Comparison: The Object (and Direction) of Misperception and Market Structure

I have shown that equilibrium per-use prices are independent of market structure. Accordingly, the comparison conducted in Section 4.D. holds, in large part, also in the monopoly case. The effect of market structure manifests in the higher base price. Utility underestimation and price overestimation reduce the difference between $p^M_x$ and $p^C_x$, since these misperceptions reduce demand and thus reduce the benefit to the monopolist from raising the base price. In contrast, utility overestimation and price underestimation
increase the difference between $p^M_x$ and $p^C_x$, since these misperceptions increase demand and thus increase the benefit to the monopolist from raising the base price.

Formally: Comparing Equations (2) and (3), and noting that $p^M_y = p^C_y$, we see that:

$$p^M_x = p^C_x + \frac{1 - F\left(\vartheta(x, y)\right)}{f\left(\vartheta(x, y)\right)}$$

The first-order effect of utility underestimation and price overestimation is to increase $\vartheta(x, y)$ and reduce demand, $1 - F\left(\vartheta(x, y)\right)$. This means that utility underestimation and price overestimation reduces the difference between monopoly pricing and competitive pricing. Applying similar reasoning, we see that utility overestimation and price underestimation increases the difference between monopoly pricing and competitive pricing.

What are the welfare implications of the higher base price that the monopolist sets? Since $p^M_y(\delta) = p^C_y(\delta)$, we have:

$$W\left(p^C_x(\delta), p^C_y(\delta); \delta\right) - W\left(p^M_x(\delta), p^M_y(\delta); \delta\right) =$$

$$= \int_{\vartheta(p_y, p^C_y; \delta)}^{\vartheta(p_y, p^M_y; \delta)} \left[\left(v + u(y(p_y)) - c_x - y(p_y)c_y\right)\right] f(v)dv$$

This difference is positive with utility underestimation and price overestimation – monopoly pricing reduces welfare. With utility overestimation and price underestimation, the difference can be either positive or negative. The higher base price reduces demand. The reduced demand avoids purchases that generate a social loss, but it might also deter purchases that generate a social gain. Accordingly, with utility overestimation and price underestimation the net welfare effect of monopoly pricing is indeterminate. (See Lemma 5 and Lemma 6.)
Finally, I consider how the positive and normative implications of a price cap depend on market structure. I focus on the case of underestimation – both utility underestimation and price underestimation, where the price cap is imposed on the per-use price. This comparison is meaningful, since the pre-cap price is independent of market structure. (This is not true for the base price, which would be capped in the case of overestimation.) I begin by asking how a cap on the per-use price affects the base price and, specifically, how this effect differs between Monopoly and Competition. A decrease in the per-use price (because of the cap) reduces the seller’s revenue form the use dimension. In a competitive market, sellers will have to increase the base price to compensate for this shortfall in revenues. The same is not true in a monopolistic market: With a mild price cap, the monopolist may decide not to increase $p_x$ in response to the reduction in $p_y$. Before increasing a price, the monopolist considers the detrimental effect of the price increase on demand for its products. For the per-use price, $p_y$, this detrimental effect is moderated by the misperception (the consumer underestimates the per-use price itself or underestimates the use-level and thus the total use-based price). No such moderation exists for the base price, $p_x$. Therefore, the monopolist may decide to absorb the reduction in profit imposed by the price cap, or some of it, rather than to try and compensate by increasing the base price.

I can now state the following proposition.

**Proposition 5 (Underestimation, Effects of a Cap on the Unregulated Price):**

With both utility underestimation and price underestimation, the increase in the base price, $p_x$, as a result of a price cap, $\tilde{p}_y$, will be smaller in a monopolistic
market, as compared to a competitive market. The difference is increasing in the magnitude of the misperception.

Proof: See Appendix.

Next consider the welfare implications of the price cap and how they depend on market structure. In a competitive market, a (properly calibrated) price cap always increases welfare with both utility and price underestimation, although the scope for welfare-enhancing regulation depends on the type of misperception. In a monopolistic market, the type of misperception determines whether there is any room for regulation. With utility underestimation, the higher monopoly base price, pre-cap, inefficiently reduces demand. A cap, on the per-use price, counteracts this effect and thus increases welfare. (A cap on the base price can also increase welfare.) With price underestimation, the lower demand caused by the higher monopoly base price can efficiently offset the inflated demand caused by the misperception. Therefore, a cap that increases demand can reduce welfare.

6. Concluding Remarks

A. Misperception on Multiple Dimensions

The model studied in this paper allowed for misperception with respect to one product dimension (the Y dimension) – either misperception about the utility derived from this dimension or about the price associated with this dimension. The other product
dimension (the X dimension) was assumed to be free of misperception. What happens when both dimensions are subject to misperception?

While the model in this paper studies one misperceived dimension and one accurately perceived dimension, the analysis could be readily extended to the case where both dimensions are subject to misperception. The dimension where misperception is more severe would be the Y dimension and the dimension where misperception is less severe would be the X dimension.

The analysis, and results, would need to be reconsidered, when the number of dimensions, and specifically the number of dimensions subject to misperception, is greater than two. Consider underestimation – of utility or of price – where a price cap on the misperceived dimension was shown to increase social welfare. The benefit from imposing a price cap would be reduced, if the seller can easily find a third dimension, where the misperception is of nearly equal magnitude.

B. Multiple Misperceptions on a Single Dimension

The analysis in this paper studied the effects of each misperception – utility misperception (under- and over-estimation) and price misperception (under- and over-estimation) – separately. What happens if consumers suffer from multiple misperceptions simultaneously? A full analysis of this richer model is beyond the scope of this paper. But even without a full analysis it is clear that simultaneous misperceptions can either reinforce one another or offset one another in non-trivial ways.

For example, utility underestimation and price underestimation both push the pre-cap, per-use price, \( p_y \), up above cost. These same misperceptions, however, exert
offsetting forces on the second-best price: utility underestimation pushes the second-best price up above cost, whereas price underestimation pulls the second-best price down below cost. The policy implications are, thus, indeterminate. For example, assume that consumers are known to underestimate the per-use price in a certain market, and the lawmaker now learns that utility is also underestimated. This discovery could either strengthen the case for imposing a price cap, because the utility underestimation further increases the pre-cap price, or it could weaken the case for imposing a price cap, because the additional misperception increases the second-best price.

A different set of interactions occurs if we consider utility underestimation and price overestimation. These two misperceptions exert offsetting forces on the pre-cap, per-use price: utility underestimation pushes \( p_y \) up above cost, whereas price overestimation pulls the pre-cap price down below cost. With opposite effects on \( p_x \). They also exert offsetting forces on the second-best price: utility underestimation pushes the second-best per-use price up above cost, whereas price overestimation pulls the second-best per-use price down, below cost. With opposite effects on the second-best base price. Once again, the policy implications are ambiguous.

C. Beyond Price-Caps

The analysis in this paper and the policy implications that follow from it may apply beyond price-caps, to other, indirect forms of price regulation. Policymakers can, and do, restrict prices in other ways. For example, the CARD Act restricts sellers’ ability to reprice credit card debt based on new information regarding the probability that the cardholder will default. Lawmakers reduce prices by changing defaults and demanding
that consumers explicitly opt-into the targeted service (as with credit card overlimit fees and overdraft protection). Finally, policymakers can influence pricing by mandating conspicuous disclosure of a specific price dimension (e.g., large font, Bold face terms in the standardized credit card disclosure, the Schumer Box) or including a certain price dimension in an influential aggregate disclosure (e.g., specifying what fees are included in the “finance charge” definition, which underlies the APR disclosure). If these disclosure strategies succeed in focusing competition on the targeted price dimension, the result would be downward pressure on the regulated price dimension.

Like price caps, these alternative price-control policies often target one price dimension within a multi-dimensional pricing structure. While each policy has unique features and merits further study, the analysis in this paper should be informative – in terms of both market outcomes and social welfare.

D. Multi-Dimensional Quality and Quality Floors

This paper focused on multi-dimensional pricing and examined the implications of capping a single price in such a multi-dimensional pricing scheme. A similar analysis applies to multi-dimensional quality. For many consumer products and services, quality is measured on multiple dimensions. Consider the cellphone market. Relevant quality dimensions include the functionality of the phone itself (the handset), the scope and duration of the warranty, the reliability of the cellular service (reception, dropped calls, etc’), the accessibility and professionalism of the provider’s customer service department, the degree of protection afforded to the customer’s personal data, the efficacy and
fairness of the contractually-specified dispute resolution mechanism, etc. The level of transparency (or disclosure) about any of these features is yet another quality dimension.

And, as with price, lawmakers often target a single quality dimension for regulation. Rather than capping a certain price dimensions, lawmakers set minimal acceptable levels, or floors, for certain quality dimensions. Sellers’ ability to disclaim implied warranties is restricted by law. The Food and Drug Administration (FDA) regulates certain dimensions of pharmaceutical products. The Consumer Product Safety Commission (CPSF) specifies minimum safety requirements for certain dimensions of certain consumer products. The unconscionability doctrine is used by courts to regulate dispute resolution mechanisms. Consumer protection law imposes minimum disclosure requirements, bans certain contractual terms, mandates cancellation or withdrawal rights (in certain cases), and so on.

Like price, quality is subject to consumer misperception. Consumers might overestimate a certain quality dimension. For example, a cellphone subscriber might overestimate the coverage provided by the carrier’s network. Consumers might also misperceive the utility associated with a certain quality dimension. For example, the cellphone subscriber might underestimate the likelihood of traveling to other parts of the country and, therefore, underestimate the utility from broad cellular coverage. Quality misperception corresponds to price misperception. And utility misperception affects price and quality in a similar way. Accordingly, the positive and normative implications of quality floors, as a function of the underlying misperception, can be studied using a framework similar to the one developed in this paper.
References


Farrell, Joseph (2008), Some Welfare Analytics of Aftermarkets, working paper.


Appendix

Proof of Lemma 1

I prove parts (a) and (b) of the lemma together.

The per-use price: we take the derivative of the social welfare function, $W(p_x, p_y; \delta)$, w.r.t. $p_y$:

$$
\frac{\partial W}{\partial p_y} = -\frac{\partial \bar{v}(p_x, p_y; \delta)}{\partial p_y} \left[ \bar{v}(p_x, p_y; \delta) + u(y(p_y)) - (c_x + y(p_y)c_y) \right] f\left( \bar{v}(p_x, p_y; \delta) \right)
$$

$$
+ \int_{\bar{v}(p_x, p_y; \delta)}^{\infty} \left[ (u'(y(p_y)) - c_y) \frac{dy(p_y)}{dp_y} \right] f(v) dv
$$

The second expression in $\frac{\partial W}{\partial p_y}$, capturing the effect on infra-marginal consumers, is zero at $p_y = c_y$, since $u'(y(c_y)) - c_y = 0$.

Therefore, at $p_y = c_y$, the sign of $\frac{\partial W}{\partial p_y}$ is determined by the sign of the first expression, which captures the effect on the marginal consumers. This effect, in turn, is comprised of two components: the demand component and the value component. The effect on demand for the product is given by:

$$
\frac{d\bar{v}(p_x, p_y; \delta)}{dp_y} = \frac{dp_x}{dp_y} - \left[ \delta u'(y(p_y, \delta)) - p_y \right] \frac{dy(p_y, \delta)}{dp_y} + y(p_y, \delta) =
$$

$$
= -\frac{dy(p_y)}{dp_y} (p_y - c_y) - (y(p_y) - \bar{v}(p_y, \delta))
$$

(Since the seller’s participation constraint is binding, we have $\Pi(p_x, p_y; \delta) = 0$, or:

$p_x = p_x(p_y) = c_x - y(p_y) \cdot (p_y - c_y)$. And so $\frac{dp_x}{dp_y} = -\frac{dy(p_y)}{dp_y} (p_y - c_y) - y(p_y)$.)
At $p_y = c_y$, this derivative is: $\frac{d\vartheta(p_x,p_y,\delta)}{dp_y} = -\left(y(p_y) - \gamma(p_y, \delta)\right)$, which is negative for $\delta < 1$ and positive for $\delta > 1$. A negative $\frac{d\vartheta(p_x,p_y,\delta)}{dp_y}$ means that increasing $p_y$ above $c_y$ increases demand (i.e., more consumers will purchase the product). A positive $\frac{d\vartheta(p_x,p_y,\delta)}{dp_y}$ means that increasing $p_y$ above $c_y$ decreases demand. The intuition is straightforward: A $1$ increase in $p_y$ increases the perceived price to be paid on the use dimension by $\gamma(p_y, \delta)$, and decreases the base price by $y(p_y)$. When $\delta < 1$, the latter effect dominates and demand goes up. When $\delta > 1$, the former effect dominates and demand goes down.

The value component, $\vartheta(p_x, p_y; \delta) + u\left(y(p_y)\right) - (p_x + y(p_y)p_y)$, can be written as:

$$u\left(y(p_y)\right) - y(p_y)p_y - \left[\delta u\left(\gamma(p_y, \delta)\right) - \gamma(p_y, \delta)p_y\right]$$

(after substituting $\vartheta(p_x, p_y; \delta) = p_x + \gamma(p_y, \delta)p_y - \delta u\left(\gamma(p_y, \delta)\right)$ ). The value component is positive for $\delta < 1$ and negative for $\delta > 1$. This means that the marginal consumer gains from the purchase with underestimation and loses from the purchase with overestimation.

Combining the demand component and the value component:

- With underestimation ($\delta < 1$), the marginal consumer gains from a purchase and so we want to increase demand. This is accomplished by increasing $p_y$. At $p_y = c_y$,

$$\frac{d\vartheta(p_x,p_y,\delta)}{dp_y} < 0$$

and, since the value component is positive, $\frac{dW}{dp_y} > 0$.

- With overestimation ($\delta > 1$), the marginal consumer loses from a purchase and so we want to decrease demand. This is accomplished by increasing $p_y$. At $p_y = c_y$,

$$\frac{d\vartheta(p_x,p_y,\delta)}{dp_y} > 0$$

and, since the value component is negative, $\frac{dW}{dp_y} > 0$. 

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The base price: $p_x^*(\delta)$ can be derived from the zero-profit condition, $\Pi(p_x, p_y) = 0$:

$$p_x = c_x - y(p_y) \cdot (p_y - c_y)$$

Since $p_y^*(\delta) > c_y$ (from part (a)), the zero-profit condition implies: $p_x^*(\delta) < c_x$.

Welfare: The welfare implications follow from the preceding analysis. QED

Proof of Lemma 2

I prove parts (a) and (b) of the lemma together.

The per-use price: As in Lemma 1, the second expression in $\frac{aw}{dp_y}$ is zero at $p_y = c_y$, and the sign of $\frac{aw}{dp_y}$ is determined by the sign of the first expression, which captures the effect on the marginal consumers. And, as in Lemma 1, this effect is comprised of two components: the demand component and the value component. The effect on demand for the product is given by:

$$\frac{d\vartheta(p_x, p_y; \delta)}{dp_y} = \frac{dp_x}{dp_y} - \left[ u' \left( y(p_y, \delta) \right) - \delta p_y \right] \frac{dy(p_y, \delta)}{dp_y} + \delta y(p_y, \delta) =$$

$$= -\frac{dy(p_y)}{dp_y} (p_y - c_y) - (y(p_y) - \delta y(p_y, \delta))$$

which, at $p_y = c_y$, is negative for $\delta < 1$ and positive for $\delta > 1$.

The value component, $\vartheta(p_x, p_y; \delta) + u \left( y(p_y) \right) - (c_x + y(p_y)c_y)$, can be reduced to:

$$u \left( y(p_y) \right) - y(p_y)p_y - \left[ u \left( y(p_y, \delta) \right) - y(p_y, \delta) \delta p_y \right]$$
In contrast to Lemma 1, the value component is negative for $\delta < 1$ and positive for $\delta > 1$. This means that the marginal consumer loses from the purchase with underestimation and gains from the purchase with overestimation.

Combining the demand component and the value component:
- With underestimation ($\delta < 1$), the marginal consumer loses from a purchase and so we want to decrease demand. This is accomplished by decreasing $p_y$. At $p_y = c_y$, 
\[
\frac{d\delta(p_x,p_y;\delta)}{dp_y} < 0 \text{ and, since the value component is negative, } \frac{dw}{dp_y} < 0.
\]
- With overestimation ($\delta > 1$), the marginal consumer gains from a purchase and so we want to increase demand. This is accomplished by decreasing $p_y$. At $p_y = c_y$, 
\[
\frac{d\delta(p_x,p_y;\delta)}{dp_y} > 0 \text{ and, since the value component is positive, } \frac{dw}{dp_y} < 0.
\]

*The base price:* With $p_y^*(\delta) < c_y$, zero-profit condition implies $p_x^*(\delta) > c_x$.

*Welfare:* The welfare implications follow from the preceding analysis.

QED

**Proof of Lemma 3**

I prove parts (a) and (b) of the lemma together.

We begin by establishing the relationship between the equilibrium prices, $p_y^C$ and $p_x^C$, and costs, $c_y$ and $c_x$. First, we show that $p_y^C > c_y$ for $\delta < 1$ and that $p_y^C < c_y$ for $\delta > 1$.

Consider Equation (1) and focus on the expression $[\hat{y}(p_y;\delta) - y(p_y)]/\frac{dy(p_y)}{dp_y}$. We show that this expression is positive for $\delta < 1$ and negative for $\delta > 1$. Taking the derivative of the FOC that determines the use level, $u'(y) = p_y$, with respect to $p_y$, we obtain:
\[
\frac{dy(p_y)}{dp_y} = \frac{1}{u''} < 0. \text{ It remains to show that } \hat{y}(p_y;\delta) - y(p_y) < 0 \text{ for } \delta < 1 \text{ and that }
\]
\( y(p_y; \delta) - y(p_y) > 0 \) for \( \delta > 1 \). To see this note that \( y(p_y; \delta = 1) = y(p_y) \) and that \( d\tilde{y}(p_y, \delta) > 0 \). This last inequality follows if we take the derivative of the FOC that determines the perceived use level, \( \delta u'(\tilde{y}) = p_y \), with respect to \( \delta \): \( \frac{d\tilde{y}(p_y, \delta)}{d\delta} = -\frac{p_y}{\delta^2 u''} > 0 \). The results for the base price, that \( p_x^c < c_x \) for \( \delta < 1 \) and that \( p_x^c > c_x \) for \( \delta > 1 \), follow when the preceding results for the per-use price are plugged into Equation (2).

Next, we consider the comparison with the optimal prices \( p_y^*(\delta) \) and \( p_y^*(\delta) \). First, compare the optimal \( p_y^*(\delta) \) (see Lemma 1) to the price that will in fact be set in a competitive market, \( p_y^C \) (see Equation (1)). For \( \delta > 1 \), we saw that \( p_y^C < c_y \) and we know (from Lemma 1) that \( p_y^C(\delta) > c_y \). So we have: \( p_y^C < c_y < p_y^*(\delta) \) and, correspondingly, \( p_y^C > c_x > p_x^*(\delta) \).

For \( \delta < 1 \), consider \( \frac{dW}{dp_y} \) from the proof of Lemma 1. We saw that \( \frac{d\tilde{y}(p_x, p_y, \delta)}{d\delta} = \frac{dp_x}{dp_y} + y(p_y, \delta) \). In the competitive equilibrium, \( y(p_y, \delta) + \frac{dp_x}{dp_y} = 0 \) (from the analysis preceding Lemma 3). Therefore, \( \frac{d\tilde{y}(p_x, p_y, \delta)}{d\delta} = 0 \) at \( p_y^C \), and \( \frac{dW}{dp_y} \) reduces to:

\[
\left. \frac{dW}{dp_y} \right|_{p_y = p_y^C} = \int_{\theta(p_x, p_y; \delta)}^\infty \left[ u'(y(p_y)) - c_y \right] \frac{dy(p_y)}{dp_y} f(v)dv
\]

which is negative for all \( p_y > c_y \). This implies that the equilibrium price, \( p_y^C \), is too high for \( \delta < 1 \) (when \( p_y > c_y \)). Moving on to the base price: Does a higher \( p_y \) (as compared to the second-best optimal \( p_y^*(\delta) \)) result in a lower or higher \( p_x \) (as compared to the second-best optimal \( p_x^*(\delta) \))? The base price, \( p_x \), is a function of \( p_y \), as defined by the zero profit condition, \( \Pi(p_x, p_y) = 0 \):

\[
p_x = p_x(p_y) = c_x - y(p_y) \cdot (p_y - c_y)
\]
We take the derivative w.r.t. $p_y$:

$$\frac{dp_x}{dp_y} = - \frac{d[y(p_y)(p_y - c_y)]}{dp_y} = - \left[ y(p_y) + \frac{dy(p_y)}{dp_y} (p_y - c_y) \right]$$

$$= - \left[ 1 + \eta_{y,p} \frac{p_y - c_y}{p_y} \right] y(p_y)$$

From Assumption 1, we know that this derivative is negative. Therefore, an increase in $p_y$ results in a decrease in $p_x$.

The welfare implications follow from these comparisons. The equilibrium prices distort use decisions and encourage excessive demand.

QED

**Proof of Lemma 5**

I prove parts (a) and (b) of the lemma together.

*The per-use price (parts (a.i). and (b.i.)):* Follows from Lemma 3, since $p^M_y = p^C_y$.

*The base price (parts (a.ii). and (b.ii.)):* From Equation (3), we have:

$$p_x = c_x - y(p_y)(p_y - c_y) + \frac{1 - F(\varphi(p_x, p_y))}{f(\varphi(p_x, p_y))}$$

Underestimation increases $p^M_y$ from $p^M_y = c_y$ to $p^M_y > c_y$ (part (a)(i)). The higher per-use price increases $y(p_y)(p_y - c_y)$ (see Assumption 1) and thus pulls $p^M_x$ down, below $c_x$. But the $\frac{1 - F(\varphi(p_x, p_y))}{f(\varphi(p_x, p_y))}$ component pushes $p^M_x$ above $c_x$. Therefore, the relationship between the equilibrium price and cost is indeterminate. For similar reasons the relationship between the equilibrium price and the second–best price is also indeterminate.
Overestimation reduces $p^*_y$ from $p^*_y = c_y$ to $p^*_y < c_y$ (part (b)(i)). Therefore, $y(p_y)(p_y - c_y) < 0$ pushing $p^*_x$ up, above $c_x$. The $\frac{1-F(\theta(p_x;p_y))}{I(\theta(p_x;p_y))}$ component pushes $p^*_x$ further above $c_x$.

**Welfare:** The welfare cost can be divided into two components:

1. The welfare cost sustained with competitive pricing, relative to the second-best:

$$W(p^*_y, p^*_x) - W(p^*_y = p^*_y, c_x) =$$

$$= \int_{\theta(p^*_y, p^*_x)}^{\theta(p^*_y, p^*_x)} \left[ (v + u \left(y \left(p^*_y \delta\right) - c_x - y \left(p^*_y \delta\right) c_y \right) \right] f(v)dv$$

$$- \int_{\theta(p^*_y, p^*_x)}^{\theta(p^*_y, p^*_x)} \left[ (v + u \left(y \left(p^*_y \delta\right) - c_x - y \left(p^*_y \delta\right) c_y \right) \right] f(v)dv$$

2. The added welfare cost from monopoly pricing on the base-price dimension:

$$W(p^*_y = p^*_y, p^*_x) - W(p^*_y, p^*_x) =$$

$$= \int_{\theta(p^*_y, p^*_x)}^{\theta(p^*_y, p^*_x)} \left[ (v + u \left(y \left(p^*_y \delta\right) - c_x - y \left(p^*_y \delta\right) c_y \right) \right] f(v)dv$$

$$- \int_{\theta(p^*_y, p^*_x)}^{\theta(p^*_y, p^*_x)} \left[ (v + u \left(y \left(p^*_y \delta\right) - c_x - y \left(p^*_y \delta\right) c_y \right) \right] f(v)dv$$

$$= \int_{\theta(p^*_y, p^*_x)}^{\theta(p^*_y, p^*_x)} \left[ (v + u \left(y \left(p^*_y \delta\right) - c_x - y \left(p^*_y \delta\right) c_y \right) \right] f(v)dv$$

Underestimation: The second welfare cost, the monopoly deadweight loss, is positive. To see this, note that $\bar{v}(p^*_y, p^*_x, \delta) > \bar{v}(p^*_y, p^*_x, \delta)$, since $p^*_x > p^*_x$ (from Equations (2) and (3)) and $p^*_y = p^*_y$ (see above); and that the per-purchase surplus is positive in the relevant range:

$$v + u \left(y \left(p^*_y \delta\right) - c_x - y \left(p^*_y \delta\right) c_y \right) \geq 0$$
(With utility underestimation, the actual value to the consumer exceeds the perceived value (which is non-negative; otherwise the consumer would not purchase): \( v + u \left( y(p_y) \right) - p_x - y(p_y)p_y \geq v + \delta u \left( \bar{y}(p_y; \delta) \right) - p_x - \bar{y}(p_y; \delta)p_y \).

With monopolistic pricing, \( p_x + y(p_y)p_y \geq c_x + y(p_y)c_y \), and so: \( v + u \left( y(p_y) \right) - c_x - y(p_y)c_y \geq v + u \left( y(p_y) \right) - p_x - y(p_y)p_y \geq 0 \).

Overestimation: As with underestimation, demand is lower in a monopolistic market: \( \bar{v}(p^*_y, p^*_y; \delta) > \bar{v}(p^*_y, p^*_x; \delta) \). However, with overestimation at least some of these lost purchases would have generated a net social loss.

QED

**Proof of Proposition 5**

In a competitive market,

\[
p_x(p_y) = c_x - y(p_y)(p_y - c_y)
\]

To examine the effects of a price cap that reduces \( p_y \), we consider the derivative:

\[
\frac{dp_x}{dp_y} = - \frac{d[y(p_y)(p_y - c_y)]}{dp_y} = -y(p_y) \left[ 1 + \eta_{y,p_y} \frac{p_y - c_y}{p_y} \right]
\]

which is negative (see Assumption 1).

In a monopolistic market,

\[
p_x(p_y) = c_x - y(p_y)(p_y - c_y) + A \left( \bar{v}(p_x(p_y), p_y) \right)
\]

where \( A \left( \bar{v}(p_x, p_y) \right) \equiv \frac{1-F(\bar{v}(p_x, p_y))}{f(\bar{v}(p_x, p_y))} \)

To examine the effects of a price cap that reduces \( p_y \), we consider the derivative:
\[
\frac{dp_x}{dp_y} = -\gamma(p_y) \left[ 1 + \eta_{y,p_y} \frac{p_y - c_y}{p_y} \right] + \frac{dA}{d\theta} \left[ \frac{\partial \bar{v}}{\partial p_y} + \frac{\partial \bar{v}}{\partial p_x} \frac{dp_x}{dp_y} \right]
\]

Or:
\[
\frac{dp_x}{dp_y} = \frac{1}{1 - \frac{dA}{d\theta}} \left\{ -\gamma(p_y) \left[ 1 + \eta_{y,p_y} \frac{p_y - c_y}{p_y} \right] + \frac{dA}{d\theta} \cdot \frac{\partial \bar{v}}{\partial p_y} \right\}
\]

Comparing \(\frac{dp_x}{dp_y}\) in a competitive market and \(\frac{dp_x}{dp_y}\) in a monopolistic market, we find that the effect in a monopolistic market is smaller as long as:

\[
(*) \quad \gamma(p_y) \left[ 1 + \eta_{y,p_y} \frac{p_y - c_y}{p_y} \right] \geq \frac{\partial \bar{v}}{\partial p_y}
\]

where \(\frac{\partial \bar{v}}{\partial p_y} = \bar{\gamma}(p_y, \delta)\) with utility underestimation and \(\frac{\partial \bar{v}}{\partial p_y} = \delta \bar{\gamma}(p_y, \delta)\) with price underestimation. I show that Inequality (*) holds for all \(\delta \leq 1\). At \(\delta = 1\), (*) holds with equality, since \(\bar{\gamma}(p_y, \delta = 1) = \gamma(p_y)\) (and also \(\delta \bar{\gamma}(p_y, \delta = 1) = \gamma(p_y)\)) and \(p_y = c_y\) in the absence of misperception. And, since \(\frac{\partial \bar{\gamma}(p_y, \delta)}{\partial \delta} > 0\) for utility misperception and \(\frac{\partial \delta \bar{\gamma}(p_y, \delta)}{\partial \delta} > 0\) for price misperception (see Assumption 2), it follows that Inequality (*) holds for all \(\delta \leq 1\). It also follows that the difference between \(\frac{dp_x}{dp_y}\) in a competitive market and \(\frac{dp_x}{dp_y}\) in a monopolistic market is increasing in the magnitude of the misperception.

Finally, note that when \(f'(\cdot)\) is large, in the relevant range, \(\frac{dA}{d\theta}\) will be very large (in absolute value). When significant misperception renders \(\frac{\partial \bar{v}}{\partial p_y}\) small, a large \(f'(\cdot)\) will result in a very small \(\frac{dp_x}{dp_y}\).

[To see the intuition for this result, consider the FOC w.r.t. the base price:]
\[ 1 - F \left( \psi(p_x, p_y) \right) = [p_x - c_x + y(p_y)(p_y - c_y)]f \left( \psi(p_x, p_y) \right) \]

This FOC balances two effects: (1) a marginal effect (RHS of the FOC): a $1 increase in \( p_x \) deters \( f \left( \psi(p_x, p_y) \right) \) consumers from purchasing the product, and the monopolist loses a profit of \( p_x - c_x + y(p_y)(p_y - c_y) \) on each of those consumers, and (2) an inframarginal effect (LHS of the FOC): a $1 increase in \( p_x \) increases per-customer profit by $1, multiplied by the \( 1 - F \left( \psi(p_x, p_y) \right) \) consumers who purchase the product.

A reduction in \( p_y \), forced by the price cap:

1) Decreases the marginal effect by reducing the profit lost on the deterred consumers.

The effect of a reduction in \( p_y \) on the per-customer profit is given by:

\[
\frac{\partial \pi(p_x, p_y)}{\partial p_y} = y(p_y) \left[ 1 + \eta_{y,p_y} \cdot \left( \frac{p_y - c_y}{p_y} \right) \right] > 0. \] 

Since \( \frac{\partial \pi(p_x, p_y)}{\partial p_x} = 1 \), to counteract this effect, \( p_x \) would need to be increased by \( y(p_y) \left[ 1 + \eta_{y,p_y} \cdot \left( \frac{p_y - c_y}{p_y} \right) \right] \), exactly the effect identified in a competitive market. But, as explained below, a monopolist may not want to fully counteract this marginal effect.

2) Increases the inframarginal effect, since a lower \( p_y \) means that more consumers are buying the product, i.e., the lower \( p_y \) reduces the threshold value, \( \psi \). The effect of a reduction in \( p_y \) on \( \psi \) is given by \( \frac{\partial \psi(p_x, p_y)}{\partial p_y} = \hat{\psi}(p_y, \delta) \) with utility underestimation, and by \( \frac{\partial \psi(p_x, p_y)}{\partial p_y} = \delta \hat{\psi}(p_y, \delta) \) with price underestimation. Note that \( \frac{d \hat{\psi}(p_y, \delta)}{d \delta} > 0 \) (for utility underestimation) and \( \frac{d \delta \hat{\psi}(p_y, \delta)}{d \delta} > 0 \) (for price underestimation; see Assumption 2), which means that with a high degree of underestimation (i.e., with small \( \delta \)) the effect of the price cap on \( \psi \), and on demand, will be small. This also means that a
small increase in $p_x$ is needed to counteract the effect of the price cap. With total underestimation, i.e., when $\delta = 0$, the price cap will have no effect on demand, and so there will be no need to increase $p_x$.

The monopolist must adjust the base price, $p_x$, to rebalance the FOC. An increase in $p_x$ increases the marginal effect and decreases the infra-marginal effect, thus rebalancing the FOC. But the required increase will be smaller than the increase in a competitive market. In particular, the increase in $p_x$ that would be required to counteract the marginal effect of the decrease in $p_y$ (which is the increase in a competitive market), will result in an excessive decrease in the infra-marginal effect, especially when the degree of underestimation is large (i.e., when $\delta$ is small.) And, when $f(\cdot)$ is increasing in the relevant range, the required increase in $p_x$ will be even smaller.]

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Figure 1a: Scope of Consumer-Surplus-Enhancing Price Caps for Different Types (and Directions) of Misperception
Figure 1b: Scope of Welfare-Enhancing Price Caps for Different Types (and Directions) of Misperception