APPENDIX

Mathematical Appendix

To develop the model of performance and reliance more formally, we will apply math and graphs to the example of the Waffle Shop. Figure 9.5 depicts the relationship between Xavier's expenditure and the probability that he will perform as promised. The variable x denotes Xavier's expenditure on performing; the variable p denotes the probability of performing; and $p = p(x)$ denotes the functional relationship between the variables. The probability of performing increases when Xavier spends; thus, p is an increasing function of x.

Now, we turn from Xavier's performance to Yvonne's reliance. Figure 9.6 graphs the relationship between the size of Yvonne's food order and her profits in September. By definition, profits in September equal total revenues minus total variable costs. Food orders are one cost that Yvonne can vary on short notice. To keep the example simple, we assume that she cannot vary any other costs in September. So the variable y, which denotes Yvonne's expenditure on food orders, also indicates her total variable costs for the month.

Total revenues equal Yvonne's income from selling meals in September. Her income from selling meals depends on whether she occupies the new building or the old building. If Xavier performs, then Yvonne occupies the new building on September 1 and she enjoys high revenues, as indicated in Figure 9.6 by the curve labeled $R_p(y)$. If Xavier does not perform, then Yvonne remains in the old building on September 1 and she enjoys low revenues, as indicated in Figure 9.6 by the curve labeled $R_{np}(y)$.

Figure 9.6 depicts profits, which equal the difference between total revenues and total variable costs, as the vertical distance between the appropriate total revenue curve and the total-cost curve. The appropriate total-revenue curve depends on the probability that Xavier finishes the building on time. If Xavier is certain to finish the building on time, then $R_p(y)$ is the appropriate total-revenue curve. Conversely, if Xavier is certain to finish the building late, then $R_{np}(y)$ is the appropriate total-revenue curve.

Yvonne maximizes profits by maximizing the vertical distance between the appropriate total-revenue curve and the total-cost curve. When $R_p(y)$ is the appropriate total-revenue curve, the high level of reliance denoted y_1 in Figure 9.6 maximizes Yvonne's profits. When $R_{np}(y)$ is the appropriate total revenue curve, the low level of reliance denoted y_0 in Figure 9.6 maximizes Yvonne's profits. (At both levels of reliance, the marginal cost of reliance (given by the constant slope of the line through the origin) equals the marginal revenue from reliance (given by the slope of either $R_p(y)$ or $R_{np}(y)$).)

Increasing the food order above y_0 is risky. The farther y rises above y_0 (up to the maximum y_1), the more Yvonne's profits increase if Xavier performs, and the more Yvonne's profits decrease if Xavier breaches.

FIGURE 9.5
The direct relationship between levels of precaution and the probability of performance.

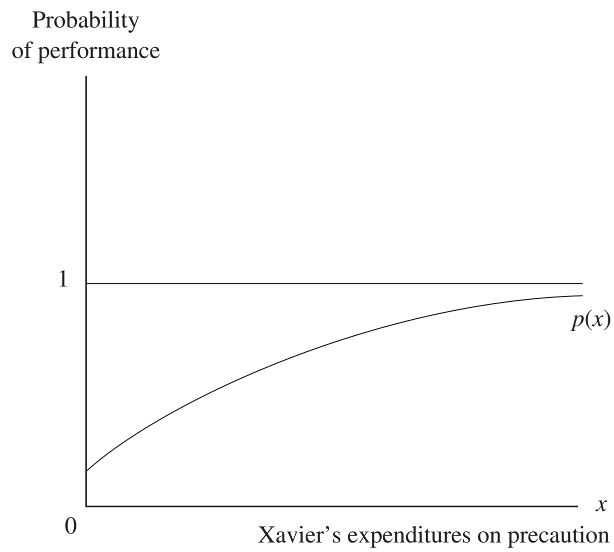
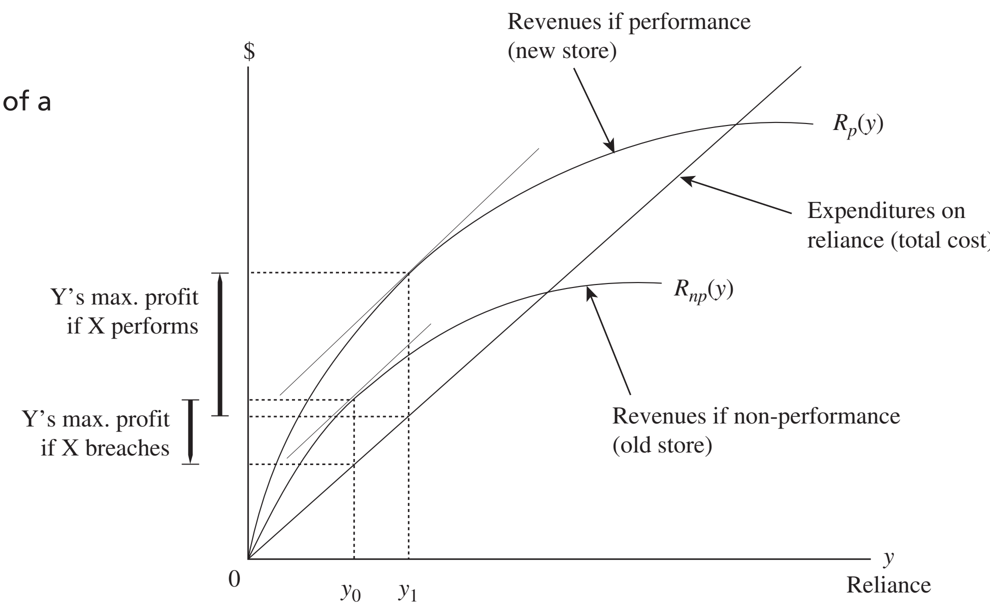


FIGURE 9.6
How a promisee's reliance depends on the probability of a promisor's performance.



The concrete example of the Waffle Shop captures two general truths. First, the promisor can take costly precautions that increase the probability that he or she will perform as promised. Second, the more the promisee relies on the promise, the greater the profits if the promise is kept, and the lower the profits if the promise is broken.

1. Efficiency Efficiency requires choosing x and y so as to maximize Yvonne's expected profits minus Xavier's expenditures. First, consider Xavier's expenditure on performance. More expenditures by Xavier increases his costs and Yvonne's expected profits, which efficiency requires Xavier to balance. Second, consider reliance. Yvonne's expenditures on reliance increase her profits if Xavier performs and decrease her profits if Xavier breaches. Efficiency requires Yvonne to balance the expected gains and losses of reliance.

We restate this verbal account of efficiency in notation. Efficiency requires choosing x and y to maximize Yvonne's expected profits minus Xavier's costs of precaution:

$$\underset{x,\,y}{\text{maximize}}\; \underbrace{\underbrace{p(x)R_p(y) + [1 - p(x)]R_{np}(y)}_{\text{Y's expected revenues}} \quad - \quad \underbrace{y}_{\text{Y's food orders}}}_{\text{Y's expected profits}} \quad - \quad \underbrace{x}_{\text{X's expenditure}} \tag{9.1}$$

We will explain how to choose *x* and *y* to maximize the preceding function. First, consider Xavier. He spends *x,* which increases the probability $p(x)$ that Yvonne enjoys high revenues equal to $R_p(y)$, rather than low revenues equal to $R_{np}(y)$. Efficiency requires the last dollar that Xavier spends to increase Yvonne's expected revenues by a dollar, which we write

$$\underbrace{1}_{\text{marginal expenditure}} = \underbrace{p'(x)[R_p(y) - R_{np}(y)]}_{\text{marginal expected revenues}} \tag{9.2}$$

(If you know calculus, note that setting the partial derivative of equation (9.1) with respect to *x* equal to zero yields equation (9.2).)

Second, consider Yvonne. Increasing her expenditure y beyond y_0 increases her revenues $R_p(y)$ with probability p and decreases her revenues $R_{np}(y)$ with probability $(1 - p)$. Efficiency requires the last dollar that Yvonne spends in reliance to increase her expected revenues by a dollar, which we write

$$\underbrace{1}_{\substack{\text{marginal reliance}\\ \text{expenditure}}} = \underbrace{pR'_p(y)}_{\substack{\text{expected increase}\\ \text{in revenues}}} + \underbrace{(1 - p)R'_{np}(y)}_{\substack{\text{possible decrease}\\ \text{in revenues}}} \tag{9.3}$$

(If you know calculus, note that setting the partial derivative of equation (9.1) with respect to *y* equal to zero yields equation (9.3).)

Equations (9.2) and (9.3) determine the values of *x* and *y* that maximize equation (9.1). These values, denoted x^* and y^*, are the efficient levels of precaution and reliance. The magnitude of y^* depends on the probability p that Xavier will perform. If performance is unlikely, then little reliance is efficient; so, the efficient value of y is close to y_0. If performance is likely, then heavy reliance is efficient, so the efficient value of y is close to y_1. If the probability of performance is greater than zero and less than 1, then y^* is in between y_0 and y_1.

2. Damages Measures Consider several different damages measures. Expectation damages D_e put Yvonne in the same position as if Xavier performed. Thus, expectation damages equal the difference between Yvonne's profits if Xavier performs, $R_p(y) - y$, and her actual profits when he breaches, $R_{np}(y) - y$. Thus, we define expectation damages:

$$\underbrace{D_e}_{\text{expectation damages}} = \underbrace{R_p(y) - R_{np}(y)}_{\text{performance revenues minus actual revenues}} \tag{9.4}$$

Opportunity-cost damages D_o put Yvonne in the same position after breach as if she had signed the best alternative contract. Yvonne signed the actual contract because she found its terms at least as good as the best alternative contract. To keep the model simple, we will say nothing explicit about the best alternative contract.

Reliance damages D_r put Yvonne in the same position after breach as if she had not signed a construction contract with Xavier or anyone else. If she had not signed a construction contract, she would have spent y_o on food and sold it in the old restaurant, thus receiving profits equal to $R_{np}(y_o) - y_o$. She actually spent y on food, Xavier breached, and she received profits equal to $R_{np}(y) - y$. The difference in profits equals her reliance damages:

$$\underbrace{D_r}_{\text{reliance damages}} = \underbrace{[R_{np}(y_o) - y_o]}_{\text{profits if no contract}} - \underbrace{[R_{np}(y) - y]}_{\text{actual profits}} \tag{9.5}$$

Now we compare the three damages measures. Performance on the contract that she actually signed is at least as good for Yvonne as performance on the best alternative contract. So, expectation damages are at least as high as opportunity-cost damages: $D_e \geq D_o$. The best alternative contract is at least as good for Yvonne as no contract. So, opportunity-cost damages are at least as high as reliance damages: $D_o \geq D_r$. In summary we have:

$$D_e \geq D_o \geq D_r \tag{9.6}$$

3. Incentives for Efficient Precaution We described efficient behavior in words and notation, and then we described alternative measures of damages. Now we consider which measure of damages creates incentives for the promisor and promisee to behave efficiently.

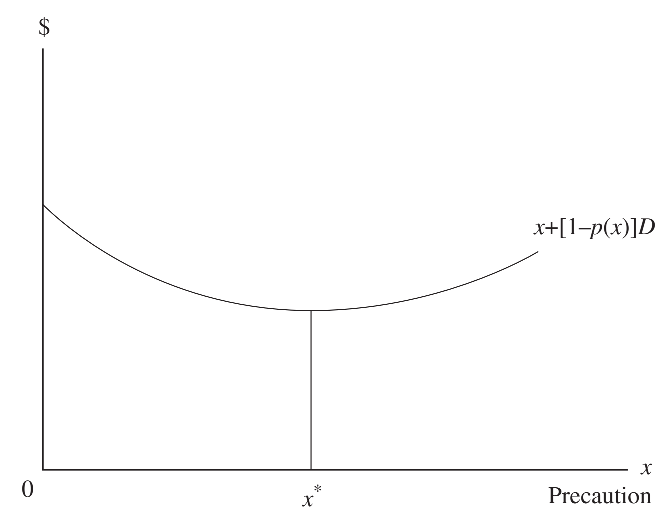
Xavier bears the full cost of his own precaution x. Xavier also bears liability for damages D with probability $[1 - p(x)]$. The sum $(x + [1 - p(x)D])$ equals Xavier's expected costs. Xavier chooses x to minimize his expected costs:

$$\underset{x}{\text{minimize}} \quad \underbrace{x}_{\text{precaution}} + \underbrace{[1 - p(x)D]}_{\text{expected liability}} \tag{9.7}$$

Figure 9.7 depicts Xavier's problem. As the figure illustrates, Xavier's costs are high if he takes no precaution because his expected damages are large. His costs are also high if he takes excessive precaution, because the precaution costs more than it saves in liability. Xavier minimizes his costs by taking precaution at an intermediate level, denoted x^* in Figure 9.7, where the expected cost curve falls to its lowest point. This occurs

FIGURE 9.7

A promisor's expected costs of precaution and of breach.



where an additional dollar spent on precaution reduces his expected liability by a dollar. In other words, his costs are minimized when the marginal cost of precaution equals the marginal reduction in expected liability:

$$\underset{\substack{\text{marginal cost}\\\text{of precaution}}}{1} = \underset{\substack{\text{marginal reduction}\\\text{in expected liability}}}{p'(x)D} \tag{9.8}$$

(If you know calculus, note that setting the partial derivative of equation (9.7) with respect to x equal to zero yields equation (9.8).)

We can compare the incentive effects of alternative measures of damages by substituting their definition for D into equation (9.8). First, consider expectation damages D_e as defined by equation (9.4). Substitute this definition of D_e for D in equation (9.8) to obtain

$$\underset{\substack{\text{marginal cost of}\\\text{precaution}}}{1} = \underset{\text{marginal expected revenues}}{p'(x)[R_p(y) - R_{np}(y)]} \tag{9.9}$$

This equation is identical to the efficiency condition in equation (9.2), which proves that expectation damages cause Xavier to take socially efficient precaution in order to minimize his expected costs.

It is easy to see why expectation damages create incentives for efficient precaution by the promisor. Promisors bear the full cost of their precaution. Their incentives are efficient when they also enjoy the full benefit. The full benefit equals any benefit that they receive plus the benefit that the promisees expect to receive. The benefit that promisees expect to receive equals the promisor's liability under expectation damages. Therefore, expectation damages cause promisors *to internalize the benefits of their precaution against breach*, which creates incentives for efficient precaution.

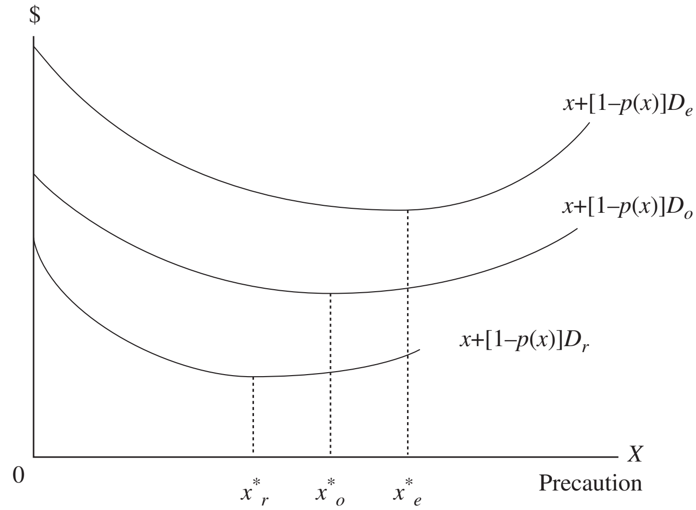
Now consider opportunity-cost damages and reliance damages. According to equation (9.6), expectation damages are at least as high as opportunity-cost damages, and opportunity-cost damages are at least as high as reliance damages. If the three damages are equal, then each of them provides incentives for efficient precaution by the promisor. If expectation damages exceed an alternative measure, then the alternative provides incentives for deficient precaution by the promisor. "Incentives for deficient precaution" means that the promisor minimizes expected costs by taking precaution below the efficient level. We summarize our conclusions as follows.

Promisor's Incentives for Precaution Against Breach

expectation		opportunity-cost		reliance
D_e	$=$	D_o	$=$	D_r
efficient		efficient		efficient
D_e	$>$	D_o	$>$	D_r
efficient		deficient		deficient

Figure 9.8 depicts these facts. Increasing the expected damages D increases Xavier's incentive to take precaution against events that cause him to breach. As damages increase from D_r to D_o, and from D_o to D_e, Xavier's cost-minimizing level of precaution increases from x_r to x_o and from x_o to x_e.

FIGURE 9.8
How precaution varies with the size of damages for breach of contract.



It is not hard to understand why awarding less than expectation damages provides incentives for deficient precaution. As explained, expectation damages cause the promisor to internalize the expected benefits of precaution. Consequently, awarding less than expectation damages causes the promisor to externalize part of the expected benefits of precaution. For example, opportunity-cost damages externalize the part of the promisee's benefit from performance of the actual contract that the promisee could not obtain from the best alternative contract.

QUESTION 9.44: Explain why perfect expectation damages generally create incentives for efficient precaution by the promisor. Explain why perfect opportunity-cost or reliance damages do not generally create incentives for efficient precaution by the promisor.

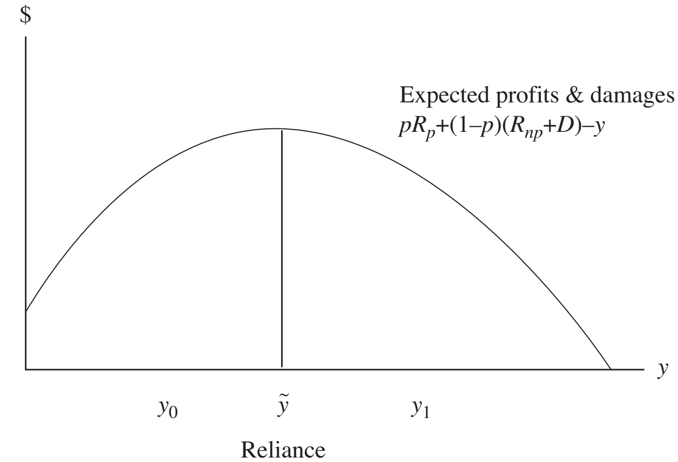
QUESTION 9.45: Assume that the remedy for breach is specific performance.

a. Use Figure 9.8 to find the amount of precaution that Xavier would take if specific performance costs the promisor the same as expectation damages.
b. Use Figure 9.8 to describe the amount of precaution that Xavier would take if specific performance costs the promisor more than expectation damages. (What are you implicitly assuming about renegotiation between the promisor and promisee?)

QUESTION 9.46: Assume that disgorgement damages are the remedy for breach, and assume that disgorgement damages exceed expectation damages. Use Figure 9.8 to describe the amount of precaution that Xavier would take.

4. Incentives for Efficient Reliance We explained that the efficiency of the promisor's incentives for precaution depend on the level of damages ("total damages"). Expectation damages provide incentives for efficient precaution by the promisor

FIGURE 9.9
Promisee's expected net profits.



against breach, whereas opportunity-cost damages and reliance damages provide deficient incentives. Now we explain how the law creates incentives for efficient reliance by the promisee. We will show that the efficiency of the promisee's incentives for reliance depends on changes in damages caused by reliance ("marginal damages").

Yvonne invests y in reliance; she receives revenues $R_p(y)$ with probability p; and she receives revenues $R_{np}(y)$ and damages D with probability $(1 - p)$. The probability-weighted sum equals her expected net profits. Yvonne chooses y to maximize her expected net profits:

$$\underset{y}{\text{maximize}} \quad \underbrace{pR_p(y) + (1 - p)(R_{np}(y) + D)}_{\text{expected revenues and damages}} \quad - \underbrace{y}_{\text{reliance}} \qquad (9.10)$$

Figure 9.9 depicts Yvonne's maximization problem. Yvonne's expected net profits are low if she does not rely ($y = 0$), because she does not order enough food in advance. Her expected net profits are also low if she relies excessively, because she orders too much food in advance. Yvonne maximizes her expected net profits by relying at an intermediate level, denoted $\tilde{y}$ in Figure 9.9, where the expected-net-profits curve reaches its highest point. This occurs where an additional dollar spent in reliance increases her expected revenues and damages by a dollar. In other words, her net profits are maximized when the marginal cost of reliance equals the marginal increase in expected revenues and damages:

$$\underbrace{1}_{\substack{\text{marginal cost}\\ \text{of reliance}}} = \underbrace{pR'_p(y) + (1 - p)R'_{np}(y)}_{\substack{\text{expected marginal}\\ \text{revenues}}} + \underbrace{(1 - p)D'}_{\substack{\text{expected marginal}\\ \text{damages}}} \qquad (9.11)$$

(If you know calculus, note that setting the partial derivative of equation (9.10) with respect to y equal to zero yields equation (9.11).)

We can compare the incentive effects of alternative measures of damages by substituting their definition for D into equation (9.10). Recall that expectation damages restore the promisee to the position that he or she would have enjoyed if the promise had been kept. In the preceding chapter we defined *perfect* expectation damages as enough money to restore the promisee to the position that he or she would have enjoyed if the promise had been kept and if reliance had been *optimal*. Applied to the Waffle Shop, perfect expectation damages equal the difference between Yvonne's revenues when Xavier performs and her revenues when he breaches, *assuming optimal reliance* ($y = y^*$):

$$\underset{\text{perfect expectation damages}}{D_e^*} = \underset{\text{expected revenues minus actual revenues, given optimal reliance}}{R_p(y^*) - R_{np}(y^*)} \qquad (9.12)$$

Notice that equation (9.12) does not contain Yvonne's actual reliance, y. It contains her optimal reliance, y^*. When reliance equals y^*, Yvonne's expected recovery of damages does not vary with her actual reliance. An additional dollar of reliance y by Yvonne does not change the damages that she receives. "Marginal damages," denoted D', means the increase in damages when Yvonne spends another dollar in reliance. Thus, if Yvonne had relied optimally, her marginal damages would equal zero: $D' = 0$. Substitute $D' = 0$ into equation (9.11) to obtain

$$\underset{\text{marginal reliance expenditure}}{1} = \underset{\text{expected increase in revenues}}{pR'_p(y)} + \underset{\text{expected decrease in revenues}}{(1-p)R'_{np}(y)} \qquad (9.13)$$

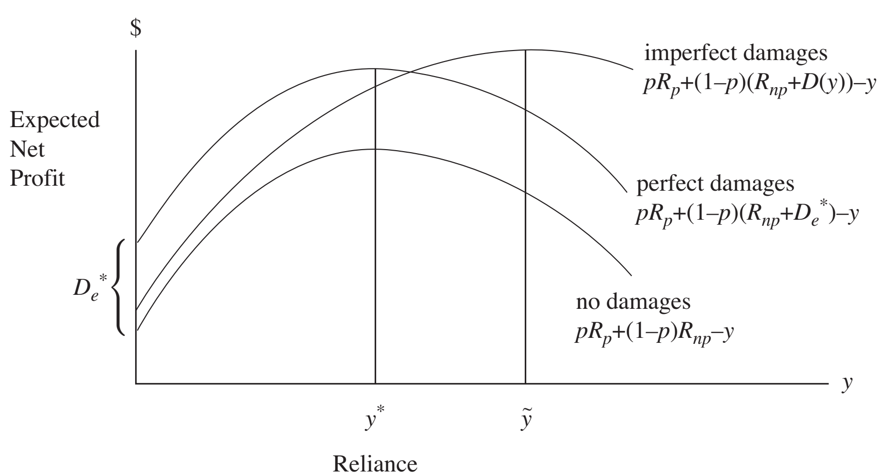
Equation (9.13) is identical to the efficiency condition in equation (9.3), which proves that perfect expectation damages cause Yvonne to rely at the socially efficient level.

It is easy to see why perfect expectation damages create incentives for efficient reliance by the promisee. Efficiency requires the person who increases risk to bear it. The promisee's reliance increases risk, specifically the risk that breach will destroy the value of the promisee's investment. Perfect expectation damages remain constant when the promisee relies more than is optimal. Thus, the risk caused by more reliance remains with the promisee. In brief, perfect expectation damages cause the promisee to internalize the risk of more than optimal reliance.

To illustrate, we contrast perfect and imperfect expectation damages in Figure 9.10. The curve labeled "no damages" indicates Yvonne's expected net profits when $D = 0$. Shift this curve up by the amount of perfect expectation damages, $D = D_e^* = D(y^*)$, to obtain the curve labeled "perfect damages." Perfect damages remain constant as reliance increases, so $D' = 0$. The curve labeled "perfect damages" in Figure 9.10 achieves its high point when Yvonne relies optimally: $y = y^*$.

FIGURE 9.10

How reliance varies with marginal damages for breach of contract.



Finally, the curve labeled "imperfect damages" in Figure 9.10 indicates Yvonne's expected net profits when damages change as a function of reliance: $D = D(y)$.[60] Notice that imperfect damages $D(y)$ increase as Yvonne's reliance y increases, so marginal damages exceed zero: $D' > 0$. This fact causes Yvonne's expected-net-profit curve to shift to the right for values of y above y^*, as depicted in Figure 9.10. As a result of the shift to the right, Yvonne's expected-net-profit curve achieves its maximum at a level of reliance, denoted $\tilde{y}$, that exceeds the efficient reliance y^*.[61] In brief, Figure 9.10 illustrates that positive marginal damages ($D' > 0$) cause overreliance ($y > y^*$).

QUESTION 9.47: Why does the "no-damages" curve achieve its maximum in Figure 9.10 for the same value of y as the "perfect-damages" curve? Explain why "no damages" provides efficient incentives for reliance by Yvonne and inefficient incentives for precaution by Xavier.

QUESTION 9.48: The "imperfect-damages" curve in Figure 9.10 lies below the "perfect-damages" curve for values of y smaller than y^*. The opposite is true for values of y larger than y^*. Consider a composite consisting of the imperfect-damages curve for values of y less than y^* and the perfect-damages curve for values of y greater than y^*:

$$D = D(y) \text{ for } y, y^*;$$
$$D = D(y^*) \text{ for } y > y^*.$$

Assume that Yvonne's expected profits correspond to this composite curve. Thus, Yvonne receives compensation for actual damages up to a maximum value of $D(y^*)$. Given this composite measure of damages, what level of reliance y maximizes Yvonne's expected profits?

QUESTION 9.49: Assume that the parties cannot renegotiate after breach. Also assume that the remedy for breach is specific performance. Specific performance guarantees that Xavier will perform. Will Yvonne set her reliance y equal to y_0, y^*, or y_1 in Figures 9.6 and 9.10? Explain your answer.

QUESTION 9.50: Assume that disgorgement damages are the remedy for breach. Disgorgement damages depend on the profits earned by the promisor as a result of breaching. Consequently, disgorgement damages do not vary with the promisee's reliance ($D' = 0$). Use Figure 9.10 to explain the incentive effects of disgorgement damages on Yvonne's reliance.

---

[60] Three facts explain the shape of the imperfect-damages curve as depicted in Figure 9.10. (1) Perfect damages exceed imperfect damages at deficient levels of reliance: $D_e^* > D(y)$ for $y < y^*$; (2) perfect damages equal imperfect damages at the efficient level of reliance: $D_e^* = D(y)$ for $y = y^*$; (3) imperfect damages exceed perfect damages for excessive levels of reliance: $D(y) > fD_e^*$ for $y > y^*$.

[61] To prove that Yvonne's reliance increases when D increases from zero to a positive number, notice that $D' > 0$ implies that the right side of equation (9.11) exceeds the efficiency condition given by equation (9.3) (and repeated in (9.13)) for any given value of y.